

# Gauge Threshold Corrections in Warped Geometry

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**Abstract.** We discuss the Kaluza-Klein threshold correction to low energy gauge couplings in theories with warped extra-dimension, which might be crucial for the gauge coupling unification when the warping is sizable. Explicit expressions of one-loop thresholds are derived for generic 5D gauge theory on a slice of  $AdS_5$ , where some of the bulk gauge symmetries are broken by orbifold boundary conditions and/or by bulk Higgs vacuum values. Effects of the mass mixing between the bulk fields with different orbifold parities are included as such mixing is required in some class of realistic warped unification models.

## 1. Introduction

In theories with unified gauge symmetry at high energy scale, threshold corrections due to heavy particles often affect the predicted low energy gauge couplings significantly [1]. Since symmetry breaking leads to a mass splitting between the particles in an irreducible representation of the unified gauge symmetry, the low energy gauge couplings generically acquire non-universal quantum corrections when the heavy particles are integrated out. In four-dimensional (4D) theories, the resulting differences between low energy gauge couplings are proportional to the logarithm of the mass ratios. Therefore, those threshold effects can be particularly important when the mass splitting occurs over a wide range of energy scales and/or for many numbers of massive particles.

Such situation can be realized in higher dimensional gauge theories (including string theories), in which there exist generically an infinite tower of gauge-charged Kaluza-Klein (KK) states. Higher dimensional gauge theories can employ a novel class of symmetry breaking mechanisms such as the one by boundary condition [2] or by the vacuum expectation value (VEV) of the extra-dimensional component of gauge field [3]. Such mechanisms might successfully address various naturalness problems of grand unified theories (GUTs) [4] and/or explain the origin of the Higgs field [5]. In higher dimensional theories with broken gauge symmetry, the whole KK tower of higher dimensional fields are splitted. This splitting can yield a large threshold correction because of the infinite number of KK modes and also a large scale difference between the lowest KK mass and the cutoff scale of the theory [6].

On the other hand, calculation of the KK thresholds requires a careful treatment of the associated UV divergences. Summing up the logarithmic contribution from each KK mode, it is expected that power-law-divergent contributions appear [7]. In field theory, all the ultraviolet (UV) divergences must be absorbed into local counterterms that are consistent with the defining symmetry of the theory. In models with unified gauge symmetry in bulk spacetime, those power-law divergences are universal and can be absorbed into a renormalization of the unified higher dimensional gauge coupling at the cutoff scale  $\Lambda$ . However, if the unified gauge symmetry is broken by a boundary condition at the orbifold fixed point, there can be non-universal logarithmically-divergent counterterms localized at the fixed point. Those logarithmic divergences are associated with the renormalization group (RG) runnings of the fixed-point gauge coupling constants [8], which lead to a controllable consequence in the predicted low-energy gauge couplings as in the case of conventional 4D GUTs [6]. After identifying the UV-divergent pieces of the KK threshold corrections, the finite calculable parts are unambiguously defined.‡ In general, those finite corrections heavily depend on the parameters of the model, including the symmetry breaking vacuum expectation values (VEVs) and the masses of higher dimensional fields, as well as on the structure of the

‡ In string theory, the full threshold corrections including stringy thresholds are finite with the cutoff scale  $\Lambda$  replaced by the string scale. For an early discussion of threshold corrections in compactified string theory, see for instance [9, 10].

background spacetime geometry.

It has been of particular interest to study quantum corrections in warped geometry. Warped extra-dimension might be responsible for the weak scale to the Planck scale hierarchy [11], or the supersymmetry breaking scale to the Planck scale hierarchy [12, 13], or even the Yukawa coupling hierarchies [14]. There also have been studies on higher dimensional GUTs in warped geometry, showing quite distinct features arising from the warping [15, 16, 17, 18, 19]. In warped models, gauge threshold corrections might be crucial for a successful unification when the lowest KK scale  $m_{KK}$  is hierarchically lower than the conventional unification scale  $M_{GUT} \sim 2 \times 10^{16}$  GeV. A series of studies on quantum corrections in anti-de Sitter space (AdS) show that KK threshold corrections in warped gauged theory is enhanced by the large logarithmic factor  $\ln(e^\Omega)$  [15, 20, 21, 22, 23], where  $e^\Omega$  is an exponentially small warp factor. Explanation of this logarithmic factor has been attempted in various contexts, including those based on the AdS/CFT correspondence which states that a 5D theory on a slice of  $AdS_5$  can be regarded as a 4D conformal field theory (CFT) with conformal symmetry spontaneously broken at  $m_{KK}$  [24, 25].

In [23], a novel method to compute 1-loop gauge couplings in higher dimensional gauge theory with warped extra dimension has been discussed, and explicit analytic expressions of the KK thresholds in 5D theory on a slice of  $AdS_5$  have been derived for the case that some part of bulk gauge symmetries are broken by orbifold boundary condition with no mass mixing between bulk fields with different orbifold parities. In this paper, we wish to extend the analysis of [23] to more general case including the possibility of symmetry breaking by bulk scalar VEVs and also of non-zero mass mixing among bulk fields with different orbifold parities. Our results then cover most of the warped GUT models discussed so far in the literatures.

The organization of this paper is as follows. In the next section, we first discuss some features of KK thresholds which are relevant for our later discussion, and then examine a simple example of 5D scalar threshold to illustrate our computation method. In Section 3, we consider generic 5D gauge theory defined on a slice of  $AdS_5$ , and derive analytic expression of 1-loop KK thresholds induced by 5D gauge and matter fields when some part of the bulk gauge symmetries are broken by orbifold boundary conditions and/or by bulk Higgs vacuum values. To be general, we also include the effects of mass mixing between the bulk fields with different orbifold parities. In Section 4, we give a conclusion. We provide in Appendix A a detailed discussion of the  $N$ -function whose zeros correspond to the KK spectrum, and a discussion of boundary matter fields in Appendix B.

## 2. Some generic features of Kaluza-Klein threshold corrections

The 5D gauge theory in consideration can be defined as a Wilsonian effective field theory with the action

$$S_W = - \int d^5x \sqrt{-G} \frac{1}{4} \left( \frac{1}{g_{5a}^2} + \frac{\kappa_a}{4\pi^2} \frac{\delta(y)}{\sqrt{G_{55}}} + \frac{\kappa'_a}{4\pi^2} \frac{\delta(y-\pi)}{\sqrt{G_{55}}} \right) F^{aMN} F_{MN}^a + S_{\text{gauge-fixing}} + S_{\text{ghost}} + S_{\text{matter}}, \quad (1)$$

where  $S_{\text{gauge-fixing}}$  and  $S_{\text{ghost}}$  are the gauge-fixing term and the associated ghost action, respectively,  $S_{\text{matter}}$  is the model-dependent action of 5D scalar and fermion matter fields, and the 5D spacetime metric  $G_{MN}$  is assumed to take a generic 4D Poincare-invariant form:

$$ds^2 = G_{MN} dx^M dx^N = e^{2\Omega(y)} \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2 \quad (0 \leq y \leq \pi),$$

where  $\pi R$  is the proper distance of the interval,  $\eta_{\mu\nu}$  corresponds to the 4D graviton zero mode which is used by low energy observer to measure the external 4D momentum  $p^\mu$  as well as the KK mass spectrum, and we are using the warp factor convention:  $e^{\Omega(y=0)} = 1$  and  $e^{\Omega(y=\pi)} \leq 1$ . Here we do not include any boundary matter field separately as it can be considered as the localized limit of bulk matter field, which is achieved by taking some mass parameters to the cutoff scale. (For a discussion of this point, see Appendix B.) Note that the range of the 5-th dimension is taken as  $0 \leq y \leq \pi$  with the convention:  $\int_0^\pi dy \delta(y) = \int_0^\pi dy \delta(y-\pi) = 1/2$ .

In order for the theory to be well-defined, one also needs to specify the UV cutoff scheme along with the Wilsonian action. Then all the Wilsonian couplings in  $S_W$  depend implicitly on the associated cutoff scheme  $\Lambda$ , and this  $\Lambda$ -dependence of Wilsonian couplings should cancel the  $\Lambda$ -dependence of regulated quantum corrections, rendering all the observable quantities to be independent of  $\Lambda$ .

The quantity of our concern is the low energy one-particle-irreducible (1PI) gauge couplings of 4D gauge boson zero modes. It can be obtained by evaluating

$$e^{i\Gamma[\Phi_{cl}]} = \int [\mathcal{D}\Phi_{qu}] e^{iS_W[\Phi_{cl} + \Phi_{qu}]} \quad (2)$$

where  $\Phi_{cl}$  denotes background field configuration which includes the 4D gauge boson zero modes  $A_\mu^{a(0)}$  as well as the vacuum values of scalar fields, and  $\Phi_{qu}$  stands for quantum fluctuations of the 5D gauge, matter and ghost fields in the model. The resulting 1PI gauge coupling  $g_a^2(p)$  of  $A_\mu^{a(0)}(p)$  carrying an external 4D momentum  $p^\mu$  is given by

$$\frac{(-p^2 \eta^{\mu\nu} + p^\mu p^\nu)}{g_a^2(p)} \equiv \frac{\delta^2 \Gamma}{\delta A_\mu^{a(0)}(p) \delta A_\nu^{a(0)}(-p)} \Big|_{A_\mu^{a(0)}=0}. \quad (3)$$

As the gauge boson zero modes have a constant wavefunction over the 5th dimension, the 4D gauge couplings at tree level are simply given by

$$\left( \frac{1}{g_a^2} \right)_{\text{tree}} = \frac{\pi R}{g_{5a}^2} + \frac{1}{8\pi^2} (\kappa_a + \kappa'_a). \quad (4)$$

To compute quantum corrections, one needs to introduce a suitable regularization scheme which might involve a set of regulator masses collectively denoted by  $\Lambda$ . One also needs to deal with a summation over the KK modes whose mass eigenvalues  $\{m_n\}$  depend on various model parameters that will be collectively denoted by  $\lambda$ , for instance the bulk or boundary masses of the matter and gauge fields as well as the AdS vacuum energy density that would determine the warp factor. Note that the 4D momentum  $p^\mu$  of the gauge boson zero modes and the KK mass eigenvalues  $\{m_n\}$  are defined in the 4D metric frame of the graviton zero mode  $\eta_{\mu\nu}$ , while  $\Lambda$ ,  $\lambda$  and  $1/R$  are the 5D mass parameters invariant under the 5D general coordinate transformation.

Schematically, one-loop correction to the 4D 1PI gauge coupling is given by

$$\frac{1}{8\pi^2}\Delta_a(p, \Lambda, R, \lambda) = \sum_{\Phi_0, \Phi_n} \int \frac{d^4l}{(2\pi)^4} f_a(p, l, m_n(R, \lambda)), \quad (5)$$

where  $\Phi_0$  denotes the light zero modes with a mass  $m_0 \ll p$ , while  $\Phi_n$  stands for the massive KK modes with  $m_n \gg p$ . In the limit  $p \ll m_{KK}$  and  $\Lambda \gg \lambda$ , where  $m_{KK}$  is the lowest KK mass, the above 1-loop correction takes the form [6, 21]

$$\begin{aligned} \frac{1}{8\pi^2}\Delta_a &= \frac{\gamma_a}{24\pi^3}\Lambda\pi R + \frac{1}{8\pi^2} \left[ \tilde{b}_a \ln(\Lambda\pi R) - b_a \ln(p\pi R) + \tilde{\Delta}_a(R, \lambda) \right] \\ &+ \mathcal{O}\left(\frac{p^2}{m_{KK}^2}\right) + \mathcal{O}\left(\frac{\lambda}{\Lambda}, \frac{1}{\Lambda R}\right). \end{aligned} \quad (6)$$

Then the low energy 1PI couplings are given by

$$\begin{aligned} \frac{1}{g_a^2(p)} &= \left(\frac{1}{g_a^2}\right)_{\text{tree}} + \frac{1}{8\pi^2}\Delta_a \\ &= \frac{\pi R}{\hat{g}_{5a}^2} + \frac{1}{8\pi^2}\hat{\kappa}_a(\ln p, \lambda, R) + \mathcal{O}\left(\frac{p^2}{m_{KK}^2}\right) + \mathcal{O}\left(\frac{\lambda}{\Lambda}, \frac{1}{\Lambda R}\right), \end{aligned} \quad (7)$$

where

$$\begin{aligned} \frac{1}{\hat{g}_{5a}^2} &= \frac{1}{g_{5a}^2(\Lambda)} + \frac{\gamma_a}{24\pi^3}\Lambda \\ \hat{\kappa}_a &= \kappa_a(\Lambda) + \kappa'_a(\Lambda) + \tilde{b}_a \ln(\Lambda\pi R) - b_a \ln(p\pi R) + \tilde{\Delta}_a(R, \lambda). \end{aligned} \quad (8)$$

The above expression of 4D 1PI coupling is valid only for  $p < m_{KK}$ . However, it still provides a well-defined matching between the observable low energy gauge couplings and the fundamental parameters in the 5D action defined at the cutoff scale  $\Lambda \gg m_{KK}$ . Note that the Wilsonian couplings  $g_{5a}^2, \kappa_a, \kappa'_a$  depend on  $\Lambda$  in such a way to make  $\hat{g}_{5a}^2$  and  $\hat{\kappa}_a$  to be independent of  $\Lambda$ .

The linearly divergent piece in (8) originates from the KK modes around the cutoff scale  $\Lambda$ , and therefore its coefficient  $\gamma_a$  severely depends on the employed cutoff scheme. For instance, in a mass-dependent cutoff scheme introducing an appropriate set of Pauli-Villars (PV) regulating fields and/or higher derivative regulating terms, each  $\gamma_a$  has a

§ At this stage, we assume a mass-dependent cutoff scheme introducing an appropriate set of Pauli-Villars regulating fields and/or higher derivative regulating terms, although eventually we will use a mass-independent dimensional regularization which is particularly convenient for the computation of gauge boson loops.

nonzero value depending on the detailed structure of the regulator masses and the regulator coefficients, while it vanishes in a mass-independent cutoff scheme such as dimensional regularization [26]]. Note that this does not affect the calculable prediction of the theory, which is determined by the scheme-independent combination  $1/\hat{g}_{5a}^2$ .

Unlike the coefficient of power-law divergence, the coefficients of  $\ln p$  and  $\ln \Lambda$  are unambiguously determined by the physics below  $\Lambda$  [6, 21]. As  $\ln p$  originates from the light zero modes with  $m_0 \ll p$ , one immediately finds

$$b_a = \frac{1}{6} \sum_{\varphi^{(0)}} \text{Tr}(T_a^2(\varphi^{(0)})) + \frac{2}{3} \sum_{\psi^{(0)}} \text{Tr}(T_a^2(\psi^{(0)})) - \frac{11}{3} \sum_{A_\mu^{(0)}} \text{Tr}(T_a^2(A_\mu^{(0)})), \quad (9)$$

where  $\varphi^{(0)}$ ,  $\psi^{(0)}$  and  $A_\mu^{(0)}$  denote the 4D real scalar, 4D chiral fermion and 4D real vector boson zero modes which originate from 5D matter and gauge fields, and  $T_a(\Phi)$  is the generator of the unbroken gauge transformation of  $\Phi$ . Note that  $\varphi^{(0)}$  can originate from a 5D vector field.

The logarithmic divergence appears because of the orbifold fixed points. This implies that  $\tilde{b}_a$  are determined just by the orbifold boundary condition of 5D fields if there is no 4D matter field confined at the fixed point. The logarithmic divergence generically takes the form

$$- \int d^5x \sqrt{-G} \frac{\ln \Lambda}{16\pi^2} \left( \frac{\lambda_{a0}\delta(y)}{\sqrt{G_{55}}} + \frac{\lambda_{a\pi}\delta(y-\pi)}{\sqrt{G_{55}}} \right) F_{\mu\nu}^a F^{a\mu\nu}, \quad (10)$$

and the coefficients  $\lambda_{a0}$  and  $\lambda_{a\pi}$  are independent of the smooth geometry of the underlying spacetime. It is then straightforward to determine  $\lambda_{a0}$  and  $\lambda_{a\pi}$  in the flat orbifold limit, which yields [6, 21]

$$\begin{aligned} \lambda_{a0} &= \sum_{zz'} \frac{z}{24} \left( \text{Tr}(T_a^2(\phi_{zz'})) - 23\text{Tr}(T_a^2(A_{zz'}^M)) \right), \\ \lambda_{a\pi} &= \sum_{zz'} \frac{z'}{24} \left( \text{Tr}(T_a^2(\phi_{zz'})) - 23\text{Tr}(T_a^2(A_{zz'}^M)) \right), \end{aligned} \quad (11)$$

and thus

$$\tilde{b}_a = \lambda_{a0} + \lambda_{a\pi} = \sum_{zz'} \frac{(z+z')}{24} \left( \text{Tr}(T_a^2(\phi_{zz'})) - 23\text{Tr}(T_a^2(A_{zz'}^M)) \right), \quad (12)$$

where  $\phi_{zz'}$  and  $A_{zz'}^M$  ( $z, z' = \pm 1$ ) denote 5D real scalar and vector fields with the orbifold boundary condition:

$$\begin{aligned} \phi_{zz'}(-y) &= z\phi_{zz'}(y), & \phi_{zz'}(-y+\pi) &= z'\phi_{zz'}(y+\pi), \\ A_{zz'}^M(-y) &= z\epsilon_M A_{zz'}^M(y), & A_{zz'}^M(-y+\pi) &= z'\epsilon_M A_{zz'}^M(y+\pi), \end{aligned} \quad (13)$$

where  $\epsilon_\mu = 1$  and  $\epsilon_5 = -1$ .

The last part of 1-loop correction, i.e.  $\tilde{\Delta}_a(R, \lambda)$ , is highly model-dependent as it generically depends on various parameters of the underlying 5D theory, e.g. the

|| A novel extension of dimensional regularization for higher dimensional gauge theory has been suggested also in [27].

curvature of background geometry, matter and gauge field masses in the bulk and at the boundaries, and also on the orbifold boundary conditions of 5D fields. Note that all of these features affect the KK mass spectrum, and thus the KK thresholds. In many cases, it can be an important part of quantum correction, even a dominant part in warped case. The aim of this paper is to provide an explicit expression of  $\tilde{\Delta}_a$  as a function of the fundamental parameters in 5D theory in a general context as much as possible.

Let us now consider a specific example of 5D scalar threshold to see some of the features discussed above. We start with the case of a massless 5D *complex* scalar field  $\phi_{zz'}$  in the flat spacetime background:

$$S_{\text{matter}} = - \int d^5x \sqrt{-G} \sum_{z,z'} G^{MN} D_M \phi_{zz'}^\dagger D_N \phi_{zz'}, \quad (14)$$

where

$$ds^2 = G_{MN} dx^M dx^N = \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2.$$

In this case, one can easily find an explicit form of the KK spectra:

$$m_n(\phi_{zz'}) = \begin{cases} \frac{n}{R} & \text{for } \phi_{++} \\ \frac{2n+1}{2R} & \text{for } \phi_{+-} \text{ and } \phi_{-+} \\ \frac{n+1}{R} & \text{for } \phi_{--}. \end{cases}, \quad (15)$$

where  $n$  is a non-negative integer. The corresponding 1-loop correction can be obtained using a simple momentum cutoff:

$$\frac{1}{8\pi^2} (p^2 \eta^{\mu\nu} - p^\mu p^\nu) \Delta_a(\phi_{zz'}) = \sum_{z,z'} \sum_{n=0}^{\Lambda R} \text{Tr}(T_a^2(\phi_{zz'})) \int \frac{d^4 l}{(2\pi)^4} f^{\mu\nu}, \quad (16)$$

where

$$f^{\mu\nu} = \frac{2\eta^{\mu\nu}((p+l)^2 + m_n^2(\phi_{zz'})) - (p+2l)^\mu(p+2l)^\nu}{i((p+l)^2 + m_n^2(\phi_{zz'}))(l^2 + m_n^2(\phi_{zz'}))},$$

which gives (in the limit  $p \ll m_{KK} = 1/R$ )

$$\begin{aligned} \Delta_a &= \frac{1}{3} \text{Tr}(T_a^2(\phi_{++})) \ln\left(\frac{\Lambda}{p}\right) \\ &+ \frac{1}{3} \left[ \text{Tr}(T_a^2(\phi_{++})) + \text{Tr}(T_a^2(\phi_{--})) \right] \sum_{n=1}^{\Lambda R} \ln\left(\frac{\Lambda R}{n}\right) \\ &+ \frac{1}{3} \left[ \text{Tr}(T_a^2(\phi_{+-})) + \text{Tr}(T_a^2(\phi_{-+})) \right] \sum_{n=1}^{\Lambda R} \ln\left(\frac{2\Lambda R}{2n-1}\right) + \mathcal{O}(1) \\ &= \frac{1}{3} \left[ \text{Tr}(T_a^2(\phi_{++})) + \text{Tr}(T_a^2(\phi_{--})) + \text{Tr}(T_a^2(\phi_{+-})) + \text{Tr}(T_a^2(\phi_{-+})) \right] \Lambda R \\ &+ \frac{1}{6} \left[ \text{Tr}(T_a^2(\phi_{++})) - \text{Tr}(T_a^2(\phi_{--})) \right] \ln(\Lambda \pi R) \\ &- \frac{1}{3} \text{Tr}(T_a^2(\phi_{++})) \ln(p\pi R) + \mathcal{O}(1). \end{aligned} \quad (17)$$

Obviously, in case with a unified gauge symmetry in bulk spacetime, the coefficients of linear divergence, i.e.  $\sum_{z,z'} \text{Tr}(T_a^2(\phi_{zz'}))$ , are universal. Also the above result gives

$$\tilde{b}_a = \frac{1}{6} \left[ \text{Tr}(T_a^2(\phi_{++})) - \text{Tr}(T_a^2(\phi_{--})) \right],$$

which confirms the result of (12). Note that  $\phi_{zz'}$  here are complex scalar fields, while  $\phi_{zz'}$  in (12) are real scalar fields.

One can generalize the above result by introducing a nonzero bulk mass. To see the effect of bulk mass, let us consider  $\phi_{++}$  with a 5D mass  $M_S \gg p$  in the flat spacetime background. It is still straightforward to find the explicit form of KK spectrum:

$$m_n = \sqrt{M_S^2 + \frac{n^2}{R^2}}. \quad (18)$$

In this case, there is no light mode since  $M_S \gg p$ , and therefore  $b_a = 0$ . Again the 1-loop threshold can be computed with a simple momentum cut off:

$$\begin{aligned} \Delta_a(\phi_{++}) &= \frac{1}{6} \text{Tr}(T_a^2(\phi_{++})) \sum_{n=0}^{\Lambda R} \ln \left( \frac{\Lambda^2 R^2}{M_S^2 R^2 + n^2} \right) + \mathcal{O}(1) \\ &= \frac{1}{6} \text{Tr}(T_a^2(\phi_{++})) \left[ 2\Lambda R + \left( \ln \frac{\Lambda}{M_S} - \ln \sinh(M_S \pi R) \right) + \mathcal{O}(1) \right]. \end{aligned}$$

For warped spacetime background, the KK spectrum takes a more complicate form, and its explicit form is usually not available. Furthermore, as the 4D loop momentum  $l^\mu$  and the KK spectrum  $\{m_n\}$  are defined in the metric frame of 4D graviton zero mode, the cutoff scales for  $l^\mu$  and  $\{m_n\}$  depend on the position in warped extra-dimension. One can avoid these difficulties using the Pole function method with dimensional regularization [26, 23, 28], which will be described below. As the 1-loop correction takes the form:

$$\frac{1}{8\pi^2} \Delta_a = \sum_{n=0}^{\infty} \int \frac{d^4 l}{(2\pi)^4} f_a(p, l, m_n), \quad (19)$$

where  $f_a \rightarrow 1/(l^2 + m_n^2)^2$  in the limit  $l^2 \sim m_n^2 \rightarrow \infty$ , one can introduce a meromorphic pole function:

$$P(q) = \frac{1}{2} \sum_{n=0}^{\infty} \left( \frac{1}{q - m_n} + \frac{1}{q + m_n} \right) \quad (20)$$

with which

$$\frac{1}{8\pi^2} \Delta_a = \int_{\Leftarrow} \frac{dq}{2\pi i} \frac{d^4 l}{(2\pi)^4} P(q) f_a(p, l, q), \quad (21)$$

where the integration contour  $\Leftarrow$  is illustrated as  $C_1$  in Fig.1. This pole function has the following asymptotic behavior at  $|q| \rightarrow \infty$ :

$$P(q) \rightarrow \frac{A}{q} + B\epsilon(\text{Im}(q)) + \mathcal{O}(q^{-2}), \quad (22)$$

where  $\epsilon(x) = x/|x|$ , and  $A$  and  $iB$  are real constants. With simple dimensional analysis, one easily finds that  $A$  and  $B$  are associated with logarithmic divergence and linear



divergence, respectively. In particular,  $iB$  corresponds to the spectral density of the KK spectrum in the UV limit  $m_n \rightarrow \infty$ , which is common to generic 5D field  $\Phi(x, y)$  with a definite 4D spin and 4D chirality, i.e.  $\Phi = \phi(x, y)$  or  $A_\mu(x, y)$  or  $\psi_{L,R}(x, y)$  with  $\gamma_5 \psi_{L,R}(x, y) = \pm \psi_{L,R}(x, y)$ . As  $A$  is associated with the coefficient (12) of logarithmic divergence, we have  $A \propto (z + z')$ , where  $z, z' = \pm 1$  are the orbifold parities of the associated 5D field at  $y = 0, \pi$ .

One may regulate the 5D momentum integral (21) by introducing an appropriate set of 5D Pauli-Villars regulator fields and/or higher derivative regulating terms in the 5D action. However, as we eventually need to include the gauge boson loops, it is more convenient to use a dimensional regularization scheme in which

$$\begin{aligned} \frac{1}{8\pi^2} \Delta_a &= \int \frac{d^{D_5} q}{2\pi i} \frac{d^{D_4} l}{(2\pi)^4} P(q) f_a(p, l, q) \\ &= \frac{c_a}{8\pi^2} \frac{A}{(4 - D_4)} + \frac{1}{8\pi^2} \Delta_a^{\text{finite}}, \end{aligned} \quad (23)$$

where  $c_a$  is some group theory coefficient, and  $\Delta_a^{\text{finite}}$  is finite in the limit  $D_5 \rightarrow 1$  and  $D_4 \rightarrow 4$ . In this regularization scheme, the irrelevant linear divergence is simply thrown away, while the logarithmic divergence appears through  $1/(D_4 - 4)$ .

After the integration over  $l^\mu$ , the remained integration over  $q$  can be done by deforming the integration contour appropriately. For the 1-loop corrections (16) induced by 5D scalar fields, we find

$$\frac{1}{8\pi^2} \Delta_a = \int_{C_1} \frac{d^{D_5} q}{2\pi i} P(q) \mathcal{G}_a(p, q), \quad (24)$$

where

$$\mathcal{G}_a(p, q) = \frac{\text{Tr}(T_a^2(\phi))}{16\pi^2} \int_0^1 dx (1 - 2x)^2 \left( \frac{2}{4 - D_4} - \ln(q^2 + x(1 - x)p^2) \right).$$

Since it depends on  $q^2$  logarithmically,  $\mathcal{G}_a$  contains a branch cut in the complex plane of  $q$ , and we can take a branch cut line along the imaginary axis with  $q^2 + x(1 - x)p^2 < 0$ . It is then convenient to divide the Pole function into three pieces:

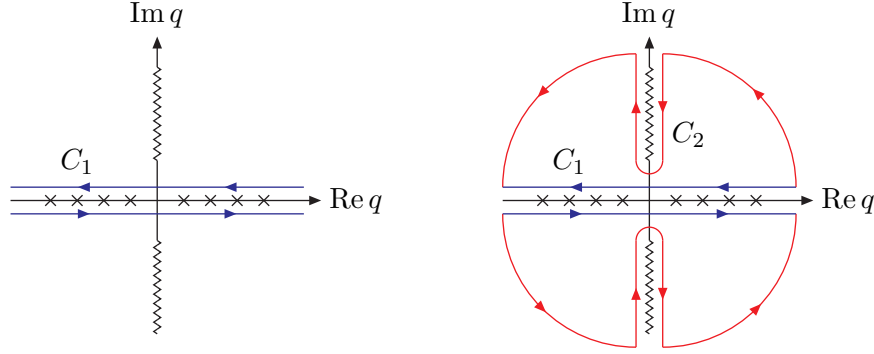
$$P(q) = \frac{A}{q} + B\epsilon(\text{Im}q) + P_{\text{finite}}(q), \quad (25)$$

where

$$P_{\text{finite}}(q) \rightarrow \mathcal{O}(q^{-2}) \quad \text{for} \quad |q| \rightarrow \infty.$$

One can then use the original contour  $C_1$  for the integration involving  $B\epsilon(\text{Im}q)$ , an infinitesimal circle around  $q = 0$  for the integration involving  $A/q$ , and finally the contour deformed as  $C_2$  in Fig.1 for the integration involving  $P_{\text{finite}}$ . Applying this procedure to the integral of the form

$$\Gamma_X = \int_{C_1} \frac{d^{D_5} q}{2\pi i} \left( \frac{A}{q} + B\epsilon(\text{Im}q) + P_{\text{finite}} \right) \left( X_0 - \ln(q^2 + X_1^2) \right),$$



**Figure 1.** Integration contours on the  $q$ -plane. Crosses along the real axis represent the KK masses  $\{\pm m_n\}$  which correspond to the poles of  $P(q)$ . The branch cut along the imaginary axis arises from  $\mathcal{G}_a(p, q)$ , and the contour  $C_1$  can be deformed to the contour  $C_2$  for the integration involving  $P_{\text{finite}}(q)$ .

one obtains

$$\begin{aligned}\Gamma_X|_{D_5 \rightarrow 1} &= A(X_0 - \ln X_1^2) - 2iB|X_1| + \int_{C_2} \frac{dq}{2\pi i} P_{\text{finite}}(q) (X_0 - \ln(q^2 + X_1^2)) \\ &= AX_0 - \ln N(q)|_{q=i|X_1|},\end{aligned}\tag{26}$$

where the  $N$ -function is defined as

$$\begin{aligned}P(q) &= \frac{1}{2} \frac{d}{dq} \ln N(q), \\ \frac{1}{2} \ln N(i|q|) &\rightarrow A \ln |q| + iB|q| \quad \text{for } |q| \rightarrow \infty.\end{aligned}\tag{27}$$

We then find the 1-loop correction due to a complex 5D scalar field is given by

$$\frac{1}{8\pi^2} \Delta_a = \frac{\text{Tr}(T_a^2(\phi))}{16\pi^2} \left[ \frac{2A}{3(4-D_4)} - \int_0^1 dx (1-2x)^2 \ln N(i\sqrt{x(1-x)p^2}) \right],$$

showing that the model-parameter dependence of low energy couplings at  $p^2 \ll m_{KK}^2$  is determined essentially by the behavior of  $N(q)$  in the limit  $q \rightarrow 0$ . For a given 5D gauge or matter field, the corresponding  $N(q)$  can be uniquely determined as will be discussed in Appendix A.

To complete the computation in dimensional regularization, one needs to subtract the  $1/(4-D_4)$  pole to define the renormalized coupling. The subtraction procedure should take into account that dimensional regularization has been applied for the momentum integral defined in the 4D metric frame of  $\eta_{\mu\nu}$ , while the correct renormalized coupling should be defined in generic 5D metric frame as a quantity invariant under the 5D general coordinate transformation. The  $1/(4-D_4)$  pole is associated with the renormalization of the fixed point gauge couplings,  $\kappa_a$  and  $\kappa'_a$ , in the action (1). For warped spacetime with

$$ds^2 = e^{2\Omega(y)} \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2 \quad (e^{\Omega(0)} = 1),$$

the logarithmic divergence structure of (10) indicates that the correct procedure is to subtract  $1/(4-D_4)$  with the counter term  $\lambda_{a0} \ln(\Lambda) + \lambda_{a\pi} \ln(\Lambda e^{\Omega(y=\pi)})$ , which would

yield

$$\Delta_a = (\lambda_{a0} + \lambda_{a\pi}) \ln \Lambda + \lambda_{a\pi} \ln (e^{\Omega(y=\pi)}) + \Delta_a^{\text{finite}}. \quad (28)$$

One can now apply the above prescription to the 1-loop correction due to a 5D complex scalar field  $\phi_{++}$  on a slice of  $AdS_5$ :

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2.$$

For  $\phi_{++}$  with a 5D mass  $M_S \gg p$ , there is no zero mode, and we find

$$\Delta_a(\phi_{++}) = \frac{1}{6} \text{Tr}(T_a^2(\phi_{++})) \left[ \ln \frac{\Lambda}{M_S} - \frac{1}{2} \ln \left( \frac{\alpha^2 - 4}{\alpha^2} \right) - \ln \sinh(\alpha \pi k R) \right],$$

where  $\alpha = \sqrt{4 + M_S^2/k^2}$ . In fact, one can get the same result using the Pauli-Villars (PV) regularization scheme in which

$$\Delta_a(\phi_{++})|_{PV} = \frac{1}{3} \text{Tr}(T_a^2(\phi_{++})) \sum_{n=0}^{\infty} \ln \left( \frac{m_n(\Phi_{++}^{\text{PV}})}{m_n(\phi_{++})} \right), \quad (29)$$

where  $m_n(\Phi_{++}^{\text{PV}})$  is the KK spectrum of the PV regulator field  $\Phi_{++}^{\text{PV}}$  which has a bulk mass  $\Lambda$ . In the limit  $n \rightarrow \infty$ ,  $m_n(\phi_{++})$  and  $m_n(\Phi_{++}^{\text{PV}})$  have the same asymptotic form  $m_n \rightarrow n\pi k/(e^{\pi k R} - 1)$ . We then have

$$\begin{aligned} \Delta_a(\phi_{++})|_{PV} &= \frac{1}{3} \text{Tr}(T_a^2(\phi_{++})) \sum_{n=0}^{\infty} \left[ \ln \left( \frac{\Lambda_0}{m_n(\phi_{++})} \right) - \ln \left( \frac{\Lambda_0}{m_n(\Phi_{++}^{\text{PV}})} \right) \right] \\ &= \Delta_a(\phi_{++})|_{DR} - \Delta_a(\Phi_{++}^{\text{PV}})|_{DR} \\ &= \frac{1}{6} \text{Tr}(T_a^2(\phi_{++})) \left[ \left( \ln \frac{\Lambda}{M_S} - \frac{1}{2} \ln \left( \frac{\alpha^2 - 4}{\alpha^2} \right) - \ln \sinh(\alpha \pi k R) \right) \right. \\ &\quad \left. + \Lambda \pi R + \mathcal{O}(1) \right], \end{aligned}$$

where  $\Lambda_0$  is an arbitrary mass parameter, the subscript DR means dimensional regularization, and the PV regulator mass is taken as  $\Lambda \gg k, 1/R$ . As we have noticed, the linearly divergent part of  $\Delta_a$  depends on the employed regularization scheme, and such a scheme-dependence can be absorbed into the renormalization of the Wilsonian 5D gauge couplings. A constant piece of order unity in  $\Delta_a$  is also scheme-dependent, and can be absorbed into the renormalization of the fixed point gauge couplings. On the other hand, the terms depending on the model parameters  $M_S, k, R$  correspond to the calculable part of  $\Delta_a$  which should be scheme-independent. The above result confirms that the two regularization schemes, DR and PV, indeed give the same calculable part of  $\Delta_a$ .

### 3. Warped gauge thresholds

In this section, we discuss the 1-loop gauge thresholds in generic 5D gauge theory on a slice of  $AdS_5$ , where some of the bulk gauge symmetries are broken by orbifold boundary

conditions and/or by bulk Higgs vacuum values. The effective action of the 4D gauge boson zero modes  $A_\mu^{a(0)}$  can be obtained by evaluating

$$e^{i\Gamma[\Phi_{cl}]} = \int [\mathcal{D}\Phi_{qu}] e^{iS_W[\Phi_{cl}+\Phi_{qu}]}, \quad (30)$$

where  $\Phi_{cl}$  denotes a background field configuration which includes  $A_\mu^{a(0)}$  as well as the Higgs vacuum values, and  $\Phi_{qu}$  stands for the quantum fluctuations of all gauge, matter and ghost fields in the model. To compute the 1-loop effective action, we need the quadratic action of those quantum fluctuations. To derive the quadratic action of  $\Phi_{qu}$ , let us start with the Wilsonian action given by

$$S_W = S_{\text{gauge}} + S_{\text{matter}} + S_{\text{gauge-fixing}} + S_{\text{ghost}}, \quad (31)$$

where

$$\begin{aligned} S_{\text{gauge}} &= - \int d^5x \sqrt{-G} \frac{1}{4} \left( \frac{1}{g_{5A}^2} + \frac{\kappa_A}{4\pi^2} \frac{\delta(y)}{\sqrt{G_{55}}} + \frac{\kappa'_A}{4\pi^2} \frac{\delta(y-\pi)}{\sqrt{G_{55}}} \right) F^{AMN} F_{MN}^A, \\ S_{\text{matter}} &= - \int d^5x \sqrt{-G} \left[ \frac{1}{2} D^M \phi^I D_M \phi^I + V(\phi) + i\bar{\psi}^p (\delta_{pq} \Gamma^M D_M + \mathcal{M}_{Fpq}(\phi)) \psi^q \right. \\ &\quad \left. + \frac{\delta(y)}{\sqrt{G_{55}}} (V_0(\phi) + 2i\mu_{pq}(\phi) \bar{\psi}^q \psi^q) - \frac{\delta(y-\pi)}{\sqrt{G_{55}}} (V_\pi(\phi) + 2i\tilde{\mu}_{pq}(\phi) \bar{\psi}^p \psi^q) \right] \end{aligned}$$

for the 5D gauge fields  $A_M^A$ , Dirac fermions  $\psi^p$ , and real scalar fields  $\phi^I$ . Here  $S_{\text{gauge-fixing}}$  is the gauge-fixing term and  $S_{\text{ghost}}$  is the associated ghost action. We fix the background spacetime to be a slice of  $\text{AdS}_5$ :

$$ds^2 = G_{MN} dx^M dx^N = e^{-2kR|y|} \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2 \quad (0 \leq y \leq \pi),$$

and impose the  $Z_2 \times Z'_2$  orbifold boundary conditions:

$$\begin{aligned} A_M^A(-y) &= z_A \epsilon_M A_M^A(y), \quad A_M^A(-y+\pi) = z'_A \epsilon_M A_M^A(y+\pi), \\ \phi^I(-y) &= z_I \phi^I(y), \quad \phi^I(-y+\pi) = z'_I \phi^I(y+\pi), \\ \psi^p(-y) &= z_p \gamma_5 \psi^p(y), \quad \psi^p(-y+\pi) = z'_p \gamma_5 \psi^p(y+\pi), \end{aligned} \quad (32)$$

where  $z_{A,I,p}, z'_{A,I,p} = \pm 1, \epsilon_\mu = 1$  and  $\epsilon_5 = -1$ . Here we ignore the boundary kinetic terms of matter fields since they are not relevant for the discussion of 1-loop gauge couplings. As for the boundary scalar potentials  $V_0$  and  $V_\pi$ , we assume for simplicity that they share (approximately) a common minimum with the bulk scalar potential  $V$ , and as a result the scalar field vacuum values are (approximately) constant along the 5-th dimension:

$$\langle \phi^I \rangle = v^I. \quad (33)$$

Then there can be two independent sources of gauge symmetry breaking, one is the bulk Higgs vacuum values  $v^I$  and the other is the orbifold boundary conditions imposed on the gauge fields.

Let us now set up the notations. In the following,  $A_M^\sigma$  denote the 5D gauge fields *not* receiving a mass from the Higgs vacuum values  $v^I$ ,  $B_M^\alpha$  are the other gauge fields which obtain a nonzero 5D mass,  $\pi^\alpha$  are the associated Goldstone bosons, and finally  $\varphi^i$

are the real-valued physical scalar field fluctuations in non-Goldstone direction. These gauge and scalar field fluctuations have the following form of the kinetic and mass terms:

$$\begin{aligned} \frac{1}{g_{5A}^2} F^{AMN} F_{MN}^A &= \frac{1}{g_{5\sigma}^2} F^{\sigma MN} F_{MN}^\sigma + \frac{1}{g_{5\alpha}^2} B^{\alpha MN} B_{MN}^\alpha, \\ D^M \phi^I D_M \phi^I &= \partial^M \varphi^i \partial_M \varphi^i + \partial^M \pi^\alpha \partial_M \pi^\alpha + \lambda_\alpha^2 B^{M\alpha} B_M^\alpha + \dots, \\ V(\phi) &= \langle V \rangle + \frac{1}{2} \mathcal{M}_{Sij}^2 \varphi^i \varphi^j + \dots, \\ V_0(\phi) &= \langle V_0 \rangle + m_{ij} \varphi^i \varphi^j + \dots, \quad V_\pi(\phi) = \langle V_\pi \rangle + \tilde{m}_{ij} \varphi^i \varphi^j + \dots, \end{aligned} \quad (34)$$

where  $F_{MN}^\sigma$  and  $B_{MN}^\alpha$  are the field strength tensor of  $A_M^\sigma$  and  $B_M^\alpha$ , respectively. Here each 5D field can have arbitrary orbifold parities, and then  $Z_2 \times Z'_2$  symmetry implies that the mass matrices take the form:

$$\begin{aligned} \mathcal{M}_{Sij}^2 &= M_{Sij}^2 \epsilon_{z_{ij} z'_{ij}}(y), \quad \mathcal{M}_{Fpq} = M_{Fpq} \epsilon_{\bar{z}_{pq} \bar{z}'_{pq}}(y), \\ m_{ij} &= m_{ij} \delta_{z_i z_j}, \quad \tilde{m}_{ij} = \tilde{m}_{ij} \delta_{z'_i z'_j}, \\ \mu_{pq} &= \mu_{pq} \delta_{\bar{z}_p \bar{z}_q}, \quad \tilde{\mu}_{pq} = \tilde{\mu}_{pq} \delta_{\bar{z}'_p \bar{z}'_q}, \end{aligned} \quad (35)$$

where  $M_{Sij}^2$  and  $M_{Fpq}$  are constant,  $z_{ij} = z_i z_j$ ,  $z'_{ij} = z'_i z'_j$ , e.t.c.,  $\bar{z} = -z$ ,  $\bar{z}' = -z'$ , and the kink function  $\epsilon_{zz'}(y)$  is defined as

$$\epsilon_{zz'}(y) = 1 \text{ for } 0 < y < \pi, \quad \epsilon_{zz'}(-y) = z \epsilon_{zz'}(y), \quad \epsilon_{zz'}(\pi - y) = z' \epsilon_{zz'}(\pi + y).$$

Note that  $M_{Sij}$ ,  $M_{Fpq}$ ,  $m_{ij}$  and  $\tilde{m}_{ij}$  have the mass dimension one, while  $\mu_{pq}$  and  $\tilde{\mu}_{pq}$  are dimensionless parameters. For a generic form of mass matrices, there can be nonzero mass mixing between matter fields with different orbifold parities. Our aim is to compute the 1-loop gauge couplings as a function of the mass parameters and the orbifold parities, which are defined above.

As we are going to compute the low energy effective action of  $A_\mu^{a(0)}(x)$ , we regard all 5D gauge fields as quantum fluctuations around a background configuration of  $A_\mu^{a(0)}(x)$  which correspond to the zero modes of  $A_M^\sigma|_{\sigma=a}$  having the orbifold parity  $z = z' = 1$ . To proceed, we choose the following form of the gauge fixing term:

$$\begin{aligned} S_{\text{gauge-fixing}} &= - \int d^5x \sqrt{-G} \left[ \frac{1}{2g_{5\sigma}^2} \left( e^{2kR|y|} \eta^{\mu\nu} D_\mu A_\nu^\sigma + \frac{1}{R^2} e^{2kR|y|} \partial_y (e^{-2kR|y|} A_5^\sigma) \right)^2 \right. \\ &\quad \left. + \frac{1}{2g_{5\alpha}^2} \left( e^{2kR|y|} \eta^{\mu\nu} D_\mu B_\nu^\alpha + \frac{1}{R^2} e^{2kR|y|} \partial_y (e^{-2kR|y|} B_5^\alpha) - g_{5\alpha}^2 \lambda_\alpha \pi^\alpha \right)^2 \right], \end{aligned} \quad (36)$$

where  $D_\mu = \partial_\mu - i A_\mu^{a(0)} T^a$  is the covariant derivative involving the background gauge boson zero modes. The corresponding ghost action is given by

$$\begin{aligned} S_{\text{ghost}} &= \int d^5x \sqrt{-G} \left[ e^{2kR|y|} \bar{c}_A^\sigma D^2 c_A^\sigma + \frac{e^{2kR|y|}}{R^2} \bar{c}_A^\sigma (\partial_y e^{-2kR|y|} \partial_y c_A^\sigma) \right. \\ &\quad \left. + e^{2kR|y|} \bar{c}_B^\alpha D^2 c_B^\alpha + \frac{e^{2kR|y|}}{R^2} \bar{c}_B^\alpha (\partial_y e^{-2kR|y|} \partial_y c_B^\alpha) - g_{5\alpha}^2 \lambda_\alpha^2 \bar{c}_B^\alpha c_B^\alpha + \dots \right], \end{aligned} \quad (37)$$

where  $c_A^\sigma$  and  $c_B^\alpha$  are the ghost fields for  $A_M^\sigma$  and  $B_M^\alpha$ , respectively, and  $D^2 = \eta^{\mu\nu} D_\mu D_\nu$ .

In the model under consideration, there are three class of field fluctuations, each of which can have arbitrary orbifold parities: (i) 5D gauge fields  $A_M^\sigma$  which do not get a mass from the Higgs vacuum values  $v^I$ , and the associated ghost fields  $c_A^\sigma$ , (ii) 5D gauge fields  $B_M^\alpha$  which get a nonzero 5D mass  $M_{V\alpha} = g_{5\alpha}\lambda_\alpha$  from  $v^I$ , and the associated Goldstone bosons and ghost fields,  $\pi^\alpha$  and  $c_B^\alpha$ , (iii) 5D Dirac fermions  $\psi^p$  and the physical scalar fields  $\varphi^i$ . After the following field redefinition,

$$\begin{aligned} A_M^\sigma &\rightarrow \frac{1}{\sqrt{R}} g_{5\sigma} A_M^\sigma, & B_M^\alpha &\rightarrow \frac{1}{\sqrt{R}} g_{5\alpha} B_M^\alpha, & \pi^\alpha &\rightarrow \frac{1}{\sqrt{R}} \pi^\alpha \\ \varphi^i &\rightarrow \frac{1}{\sqrt{R}} \varphi^i, & \psi^p &\rightarrow \frac{1}{\sqrt{R}} \psi^p, & c_A^\sigma &\rightarrow \frac{1}{\sqrt{R}} c_A^\sigma, & c_B^\alpha &\rightarrow \frac{1}{\sqrt{R}} c_B^\alpha, \end{aligned} \quad (38)$$

we find that each class of field fluctuations has the quadratic action:

$$S_2 = \int d^4x dy (\mathcal{L}_A + \mathcal{L}_B + \mathcal{L}_M), \quad (39)$$

where

$$\begin{aligned} \mathcal{L}_A &= -\frac{1}{2} \eta^{\mu\nu} A_\mu^\sigma \Delta A_\nu^\sigma - \frac{e^{-2kR|y|}}{2R^2} (\partial_y A_\mu^\sigma)^2 - \frac{e^{-2kR|y|}}{2} A_5^\sigma \Delta A_5^\sigma - \frac{e^{-4kR|y|}}{2R^2} (\partial_y A_5^\sigma)^2 \\ &\quad - \frac{1}{2} \frac{e^{-4kR|y|}}{R} (-4k^2 + 4k(\delta(y) - \delta(y - \pi))) (A_5^\sigma)^2 \\ &\quad - e^{-2kR|y|} \bar{c}_A^\sigma \Delta c_A^\sigma + \frac{e^{-2kR|y|}}{R^2} \bar{c}_A^\sigma (\partial_y e^{-2kR|y|} \partial_y c_A^\sigma) \\ \mathcal{L}_B &= -\frac{1}{2} \eta^{\mu\nu} B_\mu^\alpha \Delta B_\nu^\alpha - \frac{e^{-2kR|y|}}{2R^2} (\partial_y B_\mu^\alpha)^2 - \frac{e^{-2kR|y|}}{2} g_{5\alpha}^2 \lambda_\alpha^2 (B_5^\alpha)^2 \\ &\quad - \frac{1}{2} e^{-2kR|y|} (\pi^\alpha \Delta \pi^\alpha + B_5^\alpha \Delta B_5^\alpha) - \frac{e^{-4kR|y|}}{2R^2} ((\partial_y \pi^\alpha)^2 + (\partial_y B_5^\alpha)^2) \\ &\quad - \frac{1}{2} e^{-4kR|y|} \begin{pmatrix} \pi^\alpha & B_5^\alpha \end{pmatrix} \begin{pmatrix} g_{5\alpha}^2 \lambda_\alpha^2 & -2k g_{5\alpha} \lambda_\alpha \epsilon_{--}(y) \\ -2k g_{5\alpha} \lambda_\alpha \epsilon_{--}(y) & g_{5\alpha}^2 \lambda_\alpha^2 - 4k^2 \end{pmatrix} \begin{pmatrix} \pi^\alpha \\ B_5^\alpha \end{pmatrix} \\ &\quad - \frac{e^{-4kR|y|}}{R} 2k(\delta(y) - \delta(y - \pi)) (B_5^\alpha)^2 \\ &\quad - e^{-2kR|y|} \bar{c}_B^\alpha \Delta c_B^\alpha + \frac{e^{-2kR|y|}}{R^2} \bar{c}_B^\alpha (\partial_y e^{-2kR|y|} \partial_y c_B^\alpha) - e^{-4kR|y|} g_{5\alpha}^2 \lambda_\alpha^2 \bar{c}_B^\alpha c_B^\alpha \\ \mathcal{L}_M &= -\frac{1}{2} e^{-2kR|y|} \varphi^i \Delta \varphi^i - \frac{e^{-4kR|y|}}{2R^2} (\partial_y \varphi^i)^2 - \frac{e^{-4kR|y|}}{2} \mathcal{M}_{Sij}^2 \varphi^i \varphi^j \\ &\quad - \frac{e^{-4kR|y|}}{R} (m_{ij} \delta(y) - \tilde{m}_{ij} \delta(y - \pi)) \varphi^i \varphi^j \\ &\quad - i e^{-3kR|y|} \bar{\psi}^p \gamma^\mu D_\mu \psi^p - i \frac{e^{-2kR|y|}}{R} \bar{\psi}^p \gamma_5 (\partial_y e^{-2kR|y|} \psi^p) - i e^{-4kR|y|} \mathcal{M}_{Fpq} \bar{\psi}^p \psi^q \\ &\quad - i \frac{e^{-4kR|y|}}{R} (2\mu_{pq} \delta(y) - 2\tilde{\mu}_{pq} \delta(y - \pi)) \bar{\psi}^p \psi^q. \end{aligned}$$

Here the gauge-covariant operator  $\Delta$  is defined as

$$\Delta \Phi = (-\eta^{\mu\nu} D_\mu D_\nu + F_{\mu\nu}^{(0)} \mathcal{J}_j^{\mu\nu}) \Phi, \quad (40)$$

where  $F_{\mu\nu}^{(0)} = F_{\mu\nu}^{a(0)} T^a$  is the field strength of the gauge boson zero modes  $A_\mu^{a(0)}$ , and  $\mathcal{J}_j^{\mu\nu}$  is the 4D Lorentz generator for a field with 4D spin  $j$ , which is normalized as

$\text{tr}(\mathcal{J}_j^{\mu\nu} \mathcal{J}_j^{\rho\lambda}) = C(j)(\eta^{\mu\rho}\eta^{\nu\lambda} - \eta^{\mu\lambda}\eta^{\nu\rho})$ , where  $C(j) = (0, 1/2, 2)$  for  $j = (0, 1/2, 1)$ . Here and in the following,  $\Phi(x, y)$  stands for a 5D field which has a definite value of 4D spin  $j$  and also of 4D chirality, e.g.  $\Phi = A_\mu(x, y)$  with  $j = 1$ ,  $\Phi = \psi_{L,R}(x, y)$  with  $j = 1/2$  and  $\gamma_5 \psi_{L,R} = \pm \psi_{L,R}$ ,  $\Phi = A_5(x, y)$  or  $\varphi(x, y)$  or  $c_A(x, y)$  with  $j = 0$ . Note that the AdS curvature  $k$  generates a mixing between  $B_5^\alpha$  and  $\pi^\alpha$  in the quadratic action. Since the Goldstone boson  $\pi^\alpha$  has the same orbifold parity as  $B_\mu^\alpha$ , this is a mixing between 4D scalar fields with opposite orbifold parities.

With the quadratic action (39), the 1-loop effective action of the gauge boson zero modes is given by

$$\begin{aligned} \Gamma[A_\mu^{(0)}] = & -\frac{\pi R}{4g_{5a}^2} \int d^4x F_{\mu\nu}^{a(0)} F^{a(0)\mu\nu} \\ & + \frac{i}{2} \left( \text{Tr}_{A_\mu} \ln \tilde{\Delta}_{A_\mu} + \text{Tr}_{A_5} \ln \tilde{\Delta}_{A_5} - 2\text{Tr}_{c_A} \ln \tilde{\Delta}_{c_A} \right) \\ & + \frac{i}{2} \left( \text{Tr}_{B_\mu} \ln \tilde{\Delta}_{B_\mu} + \text{Tr}_{B_5, \pi} \ln \tilde{\Delta}_{B_5, \pi} - 2\text{Tr}_{c_B} \ln \tilde{\Delta}_{c_B} \right) \\ & + \frac{i}{2} \left( \text{Tr}_\varphi \ln \tilde{\Delta}_\varphi - \text{Tr}_{\psi_L} \ln \tilde{\Delta}_{\psi_L} - \text{Tr}_{\psi_R} \ln \tilde{\Delta}_{\psi_R} \right), \end{aligned} \quad (41)$$

where

$$\begin{aligned} \text{Tr}_\Phi \ln \tilde{\Delta}_\Phi &= \sum_n \text{Tr}_{\Phi_n} \ln (\Delta + m_n^2(\Phi)) \\ &= \int_{\equiv} \frac{d^{D_5} q}{2\pi i} P_\Phi(q) \text{Tr} \ln (\Delta + q^2). \end{aligned} \quad (42)$$

Here  $\Phi_n$  denotes the  $n$ -th KK modes with the mass eigenvalue  $m_n(\Phi)$ :

$$\Phi(x, y) = \sum_n \Phi_n(x) f_n(y), \quad (43)$$

and in the last step we have applied the Pole function technique discussed in the previous section:

$$P_\Phi(q) = \frac{1}{2} \sum_n \left( \frac{1}{q - m_n(\Phi)} + \frac{1}{q + m_n(\Phi)} \right), \quad (44)$$

where the summation includes the zero modes also.

It is straightforward to perform the integration over 4D loop momentum with dimensional regularization. We then find

$$\text{Tr} \ln (\Delta + q^2) = i \int \frac{d^4 p}{(2\pi)^4} \mathcal{G}_a^\Phi(q, p) A_\mu^a(-p) (p^2 \eta^{\mu\nu} - p^\mu p^\nu) A_\nu^a(p) + \dots,$$

where

$$\begin{aligned} \mathcal{G}_a^\Phi(q, p) &= \frac{1}{8\pi^2} \text{Tr}(T_a(\Phi)^2) \int_0^1 dx \left( \frac{1}{2} d(j_\Phi) (1 - 2x)^2 - 2C(j_\Phi) \right) \\ &\quad \times \left( \frac{2}{4 - D_4} + \ln(4\pi e^{-\gamma}) - \ln(q^2 + x(1-x)p^2) \right). \end{aligned} \quad (45)$$

Here  $d(j_\Phi) = (1, 2, 2, 4)$  and  $C(j_\Phi) = (0, 1/2, 1/2, 2)$  for  $j_\Phi = (0, 1/2_L, 1/2_R, 1)$  denoting the 4D spin and chirality of  $\Phi$ . The 1-loop correction induced by  $\Phi$  can be expressed as

$$\frac{1}{8\pi^2} \Delta_a^\Phi(p) = (-1)^{2j_\Phi} \int_{\equiv} \frac{d^{D_5} q}{2\pi i} P_\Phi(q) \mathcal{G}_a^\Phi(q, p), \quad (46)$$

where the dependence on the 4D spin and unbroken gauge charges of  $\Phi$  is encoded in  $\mathcal{G}_a^\Phi$ , while the dependence on various mass parameters is encoded in the pole function  $P_\Phi$  which contains the full information on the KK spectrum. As explained in Section 2, we can deform the integration contour appropriately to simplify the integration over  $q$ . (See Fig. 1.) Then, following the method discussed in the previous section, we find

$$\begin{aligned} \frac{1}{8\pi^2} \Delta_a^\Phi(p) &= \frac{(-1)^{2j_\Phi}}{8\pi^2} \text{Tr}(T_a^2(\Phi)) \left\{ \left( \frac{1}{12} d(j_\Phi) - C(j_\Phi) \right) \left( \frac{2A_\Phi}{4 - D_4} + \mathcal{O}(1) \right) \right. \\ &\quad \left. + \int_0^1 dx \left( C(j_\Phi) - \frac{1}{4} d(j_\Phi) (1 - 2x)^2 \right) \ln N^\Phi(i\sqrt{x(1-x)p^2}) \right\}, \end{aligned} \quad (47)$$

where the Pole function has the following asymptotic behavior at  $|q| \rightarrow \infty$ :

$$P_\Phi(q) = \frac{A_\Phi}{q} + B_\Phi \epsilon(\text{Im} q) + \mathcal{O}(q^{-2}) \quad (48)$$

and  $N^\Phi$  is a holomorphic even function define as

$$\begin{aligned} N^\Phi &= C_\Phi \prod_n (m_n^2(\Phi) - q^2) \quad \left( P_\Phi(q) = \frac{1}{2} \frac{d}{dq} \ln N^\Phi(q) \right), \\ \frac{1}{2} \ln N^\Phi(i|q|) &= A_\Phi \ln |q| + iB_\Phi |q| + \mathcal{O}(|q|^{-1}) \quad \text{at } |q| \rightarrow \infty. \end{aligned} \quad (49)$$

Since  $A_\Phi/(4 - D_4)$  is associated with the logarithmic divergence of the fixed point gauge couplings, we have  $A_\Phi \propto (z + z')$ , where  $z, z'$  are the orbifold parities of  $\Phi$ . (See Eqs.(10) and (12).) In our convention, for  $\Phi(x, y) = \{\phi, \psi_L, \psi_R, A_\mu\}$ , we have

$$A_\Phi = \frac{z + z'}{4}, \quad iB_\Phi = \frac{e^{\pi k R} - 1}{2k}, \quad (50)$$

where  $\Phi(-y) = z\Phi(y)$  and  $\Phi(-y + \pi) = z'\Phi(y + \pi)$ . Note that here  $\phi$  can be a 5D scalar, or the 5-th component of a 5D vector, or a ghost field. Also a 5D Dirac fermion  $\psi$  with orbifold parities  $z, z'$  consists of  $\psi_L$  with orbifold parities  $z, z'$  and  $\psi_R$  with orbifold parities  $\bar{z} = -z, \bar{z}' = -z'$ , and thus  $A_\psi = A_{\psi_L} + A_{\psi_R} = 0$ . As was noticed in the previous section, in warped spacetime, the renormalized fixed point gauge couplings at the cutoff scale  $\Lambda$  are obtained by subtracting the pole divergence  $(z + z')/(4 - D_4)$  with a counter term proportional to

$$\delta(y) z \ln \Lambda + \delta(y - \pi) z' \ln(e^{-k\pi R} \Lambda).$$

We are now ready to present the 1-loop corrections to low energy gauge couplings, induced by generic 5D fields on a slice of  $\text{AdS}_5$ . For this, let  $N_{zz'}^\Phi$  denote the  $N$ -function of  $\Phi(x, y)$  having a definite value of 4D spin  $j_\Phi$ , of 4D chirality, and of orbifold parities  $z, z'$ . Explicit forms of  $N_{zz'}^\Phi(q)$  and their limiting behaviors at  $|q| \rightarrow 0, \infty$  for  $\Phi$ 's with generic bulk and boundary masses are presented in Appendix A. Also let  $\{\Phi\}$  denote a set of  $\Phi$ 's having the same  $j_\Phi$  and unbroken gauge charges, but not necessarily the same orbifold parities, which generically have a mixing to each other in the quadratic action (39) of quantum fluctuations, and  $N^{\{\Phi\}}$  denote the  $N$ -function of this set of  $\Phi$ 's. Then the full 1-loop corrections are summarized as

$$\frac{1}{8\pi^2} \Delta_a = \frac{1}{8\pi^2} \left[ \Delta_a^{\{A\}} + \Delta_a^{\{B\}} + \Delta_a^{\{\psi_L\}} + \Delta_a^{\{\psi_R\}} + \Delta_a^{\{\varphi\}} \right], \quad (51)$$



where

$$\begin{aligned}\Delta_a^{\{A\}} &= \Delta_a^{\{A_\mu\}}(p) + \Delta_a^{\{A_5\}}(p) - 2\Delta_a^{\{c_A\}}(p), \\ \Delta_a^{\{B\}} &= \Delta_a^{\{B_\mu\}}(p) + \Delta_a^{\{B_5, \pi\}}(p) - 2\Delta_a^{\{c_B\}}(p)\end{aligned}\quad (52)$$

for  $\Delta_a^{\{\Phi\}}(p)$  given by

$$\begin{aligned}\Delta_a^{\{\Phi\}}(p) &= a(j_\Phi) \text{Tr}(T_a^2(\Phi)) \left[ n_0^{\{\Phi\}} \ln p - \frac{1}{2} (n_{++}^{\{\Phi\}} - n_{--}^{\{\Phi\}}) \ln \Lambda \right. \\ &\quad \left. + \frac{1}{2} \ln N_0^{\{\Phi\}} + \frac{1}{4} (n_{++}^{\{\Phi\}} - n_{+-}^{\{\Phi\}} + n_{-+}^{\{\Phi\}} - n_{--}^{\{\Phi\}}) \pi k R + \mathcal{O}(1) \right].\end{aligned}\quad (53)$$

Here  $n_0^{\{\Phi\}}$  denotes the number of zero modes in  $\{\Phi\}$ ,  $n_{zz'}^{\{\Phi\}}$  is the number of  $\Phi$ 's with orbifold parities  $z, z'$  defined as

$$\Phi(-y) = z\Phi(y), \quad \Phi(-y + \pi) = z'\Phi(y + \pi),$$

and

$$\begin{aligned}a(j_\Phi) &= \left( -\frac{1}{6}, -\frac{2}{3}, \frac{10}{3} \right) \quad \text{for } j_\Phi = \left( 0, \frac{1}{2}, 1 \right), \\ N^{\{\Phi\}}(q) &= (-q^2)^{n_0^{\{\Phi\}}} \left( N_0^{\{\Phi\}} + \mathcal{O}(q^2/m_{KK}^2) \right),\end{aligned}\quad (54)$$

where  $m_{KK}$  denotes the lightest KK mass of  $\{\Phi\}$ . The above result shows that the model-parameter dependence of 1-loop gauge couplings is determined mostly by the behavior of  $N$ -functions at  $|q| \ll m_{KK}$ , particularly by  $N_0^{\{\Phi\}}$ .

The 1-loop corrections induced by 5D Dirac fermions  $\{\psi^p\}$  take a simpler form. As the equation of motion for  $\psi$  involves  $\gamma_5$ , it is convenient to split each  $\psi$  into two chiral fermions:  $\psi = \psi_L + \psi_R$  with  $\gamma_5 \psi_{L,R} = \pm \psi_{L,R}$ , and then we always have

$$n_{zz'}^{\{\psi_L\}} = n_{\bar{z}\bar{z}'}^{\{\psi_R\}}, \quad n_0^{\{\psi_L\}} - n_0^{\{\psi_R\}} = n_{++}^{\{\psi_L\}} - n_{++}^{\{\psi_R\}}, \quad (55)$$

regardless of the bulk and boundary fermion masses  $\mathcal{M}_{Fpq}, \mu_{pq}$  and  $\tilde{\mu}_{pq}$ . (If there is no mass mixing between  $\psi^p$  with different orbifold parities,  $n_0^{\{\psi_L\}} = n_{++}^{\{\psi_L\}}$  and  $n_0^{\{\psi_R\}} = n_{++}^{\{\psi_R\}} = n_{--}^{\{\psi_L\}}$ .) Since  $\{\psi_L\}$  and  $\{\psi_R\}$  have the same KK mass spectrum, we also have

$$N^{\{\psi_L\}}(q) = (-q^2)^{n_0^{\{\psi_L\}} - n_0^{\{\psi_R\}}} N^{\{\psi_R\}}(q) \quad (56)$$

and thus

$$\begin{aligned}N^{\{\psi\}}(q) &= N^{\{\psi_L\}}(q) N^{\{\psi_R\}}(q) \\ &= (-q^2)^{n_0^{\{\psi_L\}} + n_0^{\{\psi_R\}}} \left[ \left( N_0^{\{\psi_L\}} \right)^2 + \mathcal{O}(q^2/m_{KK}^2) \right],\end{aligned}\quad (57)$$

where

$$N^{\{\psi_L\}}(q) = (-q^2)^{n_0^{\{\psi_L\}}} \left( N_0^{\{\psi_L\}} + \mathcal{O}(q^2/m_{KK}^2) \right)$$

in the limit  $|q| \ll m_{KK}$ . We then find the 1-loop gauge couplings induced by  $\{\psi\}$  are given by

$$\begin{aligned}\frac{1}{8\pi^2} \Delta_a^{\{\psi\}}(p) &= \frac{1}{8\pi^2} (\Delta_a^{\{\psi_L\}}(p) + \Delta_a^{\{\psi_R\}}(p)) \\ &= -\frac{1}{12\pi^2} \text{Tr}(T_a^2(\psi)) \left[ \left( n_0^{\{\psi_L\}} + n_0^{\{\psi_R\}} \right) \ln p + \ln N_0^{\{\psi_L\}} + \mathcal{O}(1) \right].\end{aligned}\quad (58)$$

In the above, the external momentum  $p^\mu$  of the gauge boson zero mode is assumed to be smaller than the lowest KK mass, justifying the use of the  $N$ -function at  $q \rightarrow 0$ . However, in certain parameter limit, there might be a KK state having a particularly light mass. For instance, the lightest KK state of a Dirac fermion  $\psi_{+-}$  with bulk mass  $M_F > k$  has a 4D mass  $m_{KK}^\psi \sim k e^{-(k+2M_F)\pi R/2}$  which can be much smaller than 1 TeV even when  $k e^{-k\pi R} \gtrsim \mathcal{O}(1)$  TeV. In such case, one needs to consider the gauge couplings at  $p > m_{KK}^\psi$ , which can be easily obtained from (53). To see this, let us consider the case with

$$m_1(\Phi) \leq m_2(\Phi) \leq \dots \leq m_n(\Phi) < p < m_{n+1}(\Phi), \quad (59)$$

in which there are  $n_0^\Phi + n$  light modes with a mass smaller than  $p$ . One can then consider the  $N$ -function at  $m_n < q < m_{n+1}$ , which can be expressed as

$$N^\Phi(q) = (-q^2)^{n_0^\Phi} \left( \prod_{l=1}^n (m_l^2 - q^2) \right) (N_n^\Phi + \mathcal{O}(q^2/m_{n+1}^2)), \quad (60)$$

where

$$N_n^\Phi = N_0^\Phi / \prod_{l=1}^n m_l^2, \quad (61)$$

and find that the 1-loop gauge couplings at  $m_n < p < m_{n+1}$  are given by

$$\begin{aligned} \frac{1}{8\pi^2} \Delta_a^\Phi(p) &= \frac{a(j_\Phi)}{8\pi^2} \text{Tr}(T_a^2(\Phi)) \left[ (n_0^\Phi + n) \ln p - \frac{1}{2} (n_{++}^\Phi - n_{--}^\Phi) \ln \Lambda \right. \\ &\quad \left. + \frac{1}{2} \ln N_n^\Phi + \frac{1}{4} (n_{++}^\Phi - n_{+-}^\Phi + n_{-+}^\Phi - n_{--}^\Phi) \pi k R + \mathcal{O}(1) \right]. \end{aligned} \quad (62)$$

As the  $N$ -functions play a crucial role in our analysis, let us discuss some relevant features of  $N^{\{\Phi\}}$  here. More complete discussion will be given in Appendix A. First, for  $A_M^\sigma = (A_\mu^\sigma, A_5^\sigma)$  and  $c_A^\sigma$  with the orbifold parities

$$\begin{aligned} A_\mu^\sigma(-y) &= z_\sigma A_\mu^\sigma(y), \quad A_5^\sigma(-y) = -z_\sigma A_5^\sigma(y) \equiv \bar{z}_\sigma A_5^\sigma(y), \\ A_\mu^\sigma(-y + \pi) &= z'_\sigma A_\mu^\sigma(y + \pi), \quad A_5^\sigma(-y + \pi) = -z'_\sigma A_5^\sigma(y + \pi) \equiv \bar{z}'_\sigma A_5^\sigma(y + \pi), \\ c_A^\sigma(-y) &= z_\sigma c_A^\sigma(y), \quad c_A^\sigma(-y + \pi) = z'_\sigma c_A^\sigma(y + \pi) \end{aligned}$$

we have

$$N_{z_\sigma z'_\sigma}^{A_\mu^\sigma}(q) = N_{z_\sigma z'_\sigma}^{c_A^\sigma}(q) = (-q^2)^{(z_\sigma + z'_\sigma)/2} N_{\bar{z}_\sigma \bar{z}'_\sigma}^{A_5^\sigma}(q). \quad (63)$$

This relation simply means that  $A_\mu^\sigma$ ,  $A_5^\sigma$  and  $c_A^\sigma$  have the same KK mass spectra, which explains the form of  $\Delta_a^{\{A\}}$  in (52). Here the factor  $q^{z_\sigma + z'_\sigma}$  represents the zero mode of  $A_\mu^\sigma$  with  $z_\sigma = z'_\sigma = 1$  or of  $A_5^\sigma$  with  $\bar{z}_\sigma = \bar{z}'_\sigma = 1$ . In the quadratic action (39),  $A_M^\sigma$  does not have any mixing with other fields, and therefore

$$N^{\{A_\mu\}} = \prod_\sigma N_{z_\sigma z'_\sigma}^{A_\mu^\sigma}, \quad N^{\{A_5\}} = \prod_\sigma N_{\bar{z}_\sigma \bar{z}'_\sigma}^{A_5^\sigma}, \quad N^{\{c_A\}} = \prod_\sigma N_{z_\sigma z'_\sigma}^{c_A^\sigma}. \quad (64)$$

On the other hand, for  $B_M^\alpha = (B_\mu^\alpha, B_5^\alpha)$  and the associated Goldstone and ghost fields,  $\pi^\alpha$  and  $c_B^\alpha$ , there is a mass mixing between  $B_5^\alpha$  and  $\pi^\alpha$  which have opposite orbifold parities. We still have

$$N_{z_\alpha z'_\alpha}^{B_\mu^\alpha}(q) = N_{z_\alpha z'_\alpha}^{c_B^\alpha}(q), \quad N^{\{B_5^\alpha, \pi^\alpha\}}(q) = N_{z_\alpha z'_\alpha}^{B_\mu^\alpha}(q) N_{\bar{z}_\alpha \bar{z}'_\alpha}^{\tilde{B}_\mu^\alpha}(q), \quad (65)$$

where  $\tilde{B}_\mu^\alpha$  is an artificial vector field which has the same bulk mass as  $B_\mu^\alpha$  and also the boundary masses given by

$$- \int d^4x dy \frac{e^{-2kR|y|}}{R} \left( 2k\delta(y) - 2k\delta(y - \pi) \right) \eta^{\mu\nu} \tilde{B}_\mu^\alpha \tilde{B}_\nu^\alpha.$$

We then find

$$\begin{aligned} N^{\{B_\mu\}} &= \prod_\alpha N_{z_\alpha z'_\alpha}^{B_\mu^\alpha}, \quad N^{\{c_B\}} = \prod_\alpha N_{z_\alpha z'_\alpha}^{c_B^\alpha}, \\ N^{\{B_5, \pi\}}(q) &= \prod_\alpha \left( N_{z_\alpha z'_\alpha}^{B_\mu^\alpha}(q) N_{\bar{z}_\alpha \bar{z}'_\alpha}^{\tilde{B}_\mu^\alpha}(q) \right). \end{aligned} \quad (66)$$

In the presence of mixing between fields with different orbifold parities,  $N^{\{\psi\}}$  and  $N^{\{\varphi\}}$  generically take a highly complicate form. Here we present the results for relatively simple cases, (i) two Dirac fermions with generic bulk and boundary masses and (ii) two scalar fields with just bulk masses, while leaving the discussion for more general case in Appendix A. Let us first consider the case of two Dirac fermions  $\{\psi_{z_p z'_p}^p\}$  ( $p = 1, 2$ ) with the following bulk and boundary masses:

$$\mathcal{M}_{Fpq}, \quad \mu_{12}, \quad \tilde{\mu}_{12}. \quad (67)$$

Note that  $\mu_{pp} = \tilde{\mu}_{pp} = 0$ , and the Dirac fermion  $\psi_{z_p z'_p}^p$  consists of  $\psi_L^p$  with orbifold parities  $z_p, z'_p$  and  $\psi_R^p$  with orbifold parities  $\bar{z}_p = -z_p, \bar{z}'_p = -z'_p$ . In the fundamental domain  $0 < y < \pi$ , the  $2 \times 2$  bulk mass matrix can be described by two mass eigenvalues  $M_{Fp}$  ( $p = 1, 2$ ) and a mixing angle  $\theta_F$ :

$$U \begin{pmatrix} M_{F11} & M_{F12} \\ M_{F21} & M_{F22} \end{pmatrix} U^\dagger = \begin{pmatrix} M_{F1} & 0 \\ 0 & M_{F2} \end{pmatrix}, \quad U = \begin{pmatrix} \cos \theta_F & \sin \theta_F \\ -\sin \theta_F & \cos \theta_F \end{pmatrix}. \quad (68)$$

Let  $N_{zz'}^{\psi_{L,R}(M)}$  denote the  $N$  function of  $\psi_{L,R}$  with orbifold parities  $z, z'$  and a bulk mass  $M$ . We then find that the  $N$ -function of the above two Dirac fermions is given by

$$\begin{aligned} N^{\{\psi^1, \psi^2\}}(q) &= N^{\{\psi_L^1, \psi_L^2\}}(q) N^{\{\psi_R^1, \psi_R^2\}}(q) \\ &= (-q^2)^{-(z_1+z'_1+z_2+z'_2)/2} \left( N^{\{\psi_L^1, \psi_L^2\}}(q) \right)^2, \end{aligned} \quad (69)$$

where

$$\begin{aligned} N^{\{\psi_L^1, \psi_L^2\}}(q) &= \left( c_0 c_\pi^* N_{z_1 z'_1}^{\psi_L(M_{F1})} + s_0 s_\pi^* N_{z_1 z'_1}^{\psi_L(M_{F2})} \right) \left( s_0^* s_\pi N_{z_2 z'_2}^{\psi_L(M_{F1})} + c_0^* c_\pi N_{z_2 z'_2}^{\psi_L(M_{F2})} \right) \\ &\quad - \left( c_0 s_\pi N_{z_1 z'_2}^{\psi_L(M_{F1})} - s_0 c_\pi N_{z_1 z'_2}^{\psi_L(M_{F2})} \right) \left( s_0^* c_\pi^* N_{z_2 z'_1}^{\psi_L(M_{F1})} - c_0^* s_\pi^* N_{z_2 z'_1}^{\psi_L(M_{F2})} \right) \end{aligned} \quad (70)$$

for

$$\begin{aligned} c_0 &= \frac{\cos \theta_F - z_1 \mu_{12} \sin \theta_F}{\sqrt{1 + |\mu_{12}|^2}}, \quad c_\pi = \frac{\cos \theta_F - z'_1 \tilde{\mu}_{12} \sin \theta_F}{\sqrt{1 + |\tilde{\mu}_{12}|^2}} \\ s_0 &= \frac{\sin \theta_F + z_1 \mu_{12} \cos \theta_F}{\sqrt{1 + |\mu_{12}|^2}}, \quad s_\pi = \frac{\sin \theta_F + z'_1 \tilde{\mu}_{12} \cos \theta_F}{\sqrt{1 + |\tilde{\mu}_{12}|^2}}. \end{aligned} \quad (71)$$

Note that this  $N$ -function takes a factorized form,  $N^{\{\psi_L^1, \psi_L^2\}} = N_{zz'}^{\psi_L(M_{F1})} N_{zz'}^{\psi_L(M_{F2})}$  if  $\psi^1$  and  $\psi^2$  have the same orbifold parities. One can similarly get the  $N$ -function of two

scalar field system  $\{\varphi_{z_i z'_i}^i\}$  ( $i = 1, 2$ ) with a generic form of the bulk mass matrix  $\mathcal{M}_{Sij}^2$  and no boundary masses. Again,  $\mathcal{M}_{Sij}^2$  can be described by two mass-square eigenvalues  $M_{Si}^2$  ( $i = 1, 2$ ) and a mixing angle  $\theta_S$ . Then the  $N$ -function of  $\{\varphi^i\}$  is given by

$$N^{\{\varphi^1, \varphi^2\}}(q) = \left( c^2 N_{z_1 z'_1}^{\varphi(M_{S1})} + s^2 N_{z_1 z'_1}^{\varphi(M_{S2})} \right) \left( s^2 N_{z_2 z'_2}^{\varphi(M_{S1})} + c^2 N_{z_2 z'_2}^{\varphi(M_{S2})} \right) - c^2 s^2 \left( N_{z_1 z'_2}^{\varphi(M_{S1})} - N_{z_1 z'_2}^{\varphi(M_{S2})} \right) \left( N_{z_2 z'_1}^{\varphi(M_{S1})} - N_{z_2 z'_1}^{\varphi(M_{S2})} \right), \quad (72)$$

where  $c = \cos \theta_S$ ,  $s = \sin \theta_S$  and  $N_{zz'}^{\varphi(M)}$  is the  $N$  function of a 5D scalar with orbifold parities  $z, z'$ , which has a bulk mass  $M$  and vanishing boundary masses.

In Appendix A, we provide explicit expression of  $N_{zz'}^\Phi$  for  $\Phi$  with various 4D spin and orbifold parities, as well as its limiting behaviors at  $|q| \rightarrow 0, \infty$ . Once the  $N$ -functions are obtained, one can examine the behavior at  $q \rightarrow 0$  to find  $N_0^{\{\Phi\}}$ , and finally apply (53) to obtain the 1-loop corrections  $\Delta_a$ . Using the properties of  $N$ -functions described above and also in Appendix A, we find the expressions of  $\Delta_a^{\{A\}}$  and  $\Delta_a^{\{B\}}$  presented in Table 1 and Table 2, respectively. (See (52) for the definition of  $\Delta_a^{\{A\}, \{B\}}$ .) For the 1-loop corrections  $\Delta_a^{\{\varphi\}, \{\psi\}}$  induced by scalar and fermion fields, we consider the two cases: one for the case that there is no mixing between matter fields with different orbifold parities, and the other case with two scalars or two Dirac fermions which can have such a mixing. For the first case, one can simply consider a single scalar or a single fermion with definite orbifold parities, and the results are summarized in Table 3 and Table 4. For the second case, one can use the  $N$ -functions (70) and (72) to obtain the results presented in Table 5 and 6. A prescription for  $\Delta_a^{\{\varphi\}, \{\psi\}}$  in more general case is described in Appendix A.

#### 4. Conclusion

Models with warped extra dimension might provide an explanation for various puzzles in particle physics, e.g. the weak scale to Planck scale hierarchy and the Yukawa coupling hierarchy, while implementing a breaking of unified gauge symmetry in bulk spacetime by boundary conditions, which would solve some of the naturalness problems in grand unified theories such as the doublet-triplet splitting problem. Kaluza-Klein threshold corrections in such models are generically enhanced by the logarithm of an exponentially small warp factor, and therefore can be crucial for successful gauge coupling unification in the framework of warped unified model. In this paper, we discuss a novel method to compute 1-loop gauge couplings in generic 5D gauge theory on a slice of  $AdS_5$ , in which some of the bulk gauge symmetries are broken by orbifold boundary conditions and/or by bulk Higgs vacuum values, and also there can be nonzero mass mixings between the bulk fields with different orbifold parities. Explicit analytic expressions of the Kaluza-Klein thresholds as a function of various model parameters are derived, and our analysis can cover most of the warped GUT models which have been discussed so far in the literatures.

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## Appendix A. $N$ function

In this appendix, we discuss the  $N$  function of a 5D field  $\Phi$  on a slice of  $\text{AdS}_5$ , which has a definite value of 4D spin and 4D chirality as well as definite orbifold parities:

$$\Phi(x, y) = \{\phi, e^{-2kR|y|}\psi_L, e^{-2kR|y|}\psi_R, A_\mu\} \quad (\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi),$$

where the 4D scalar  $\phi$  might be a 5D scalar, or the 5-th component of a 5D vector, or a ghost field. Generic 5D field  $\Phi$  on a slice of  $\text{AdS}_5$  can be decomposed as

$$\Phi(x, y) = \sum \Phi_n(x) f_n(y), \quad (\text{A.1})$$

where the KK wavefunction  $f_n$  satisfies

$$[-e^{skR|y|}\partial_y (e^{-skR|y|}\partial_y) + R^2 M_\Phi^2] f_n = R^2 e^{2kR|y|} m_n^2 f_n \quad (\text{A.2})$$

for the KK mass eigenvalue  $m_n$ . Here

$$M_\Phi^2 = \{M_S^2, M_F(M_F + k), M_F(M_F - k), M_V^2\}, \\ s = \{4, 1, 1, 2\} \quad \text{for} \quad \Phi = \{\phi, e^{-2kR|y|}\psi_L, e^{-2kR|y|}\psi_R, A_\mu\}, \quad (\text{A.3})$$

where  $M_S, M_F$  and  $M_V$  denote the bulk masses of  $\phi, \psi$  and  $A_\mu$ , respectively. Here we are using the mass parameter convention defined in (34) and (35), e.g.  $M_S = 0$  for  $\phi = A_5^\sigma$  or  $c_A^\sigma$ ,  $M_S = g_{5\alpha}\lambda_\alpha$  for  $\phi = c_B^\alpha$ ,  $M_V = 0$  for  $A_\mu = A_\mu^\sigma$ , and  $M_V = g_{5\alpha}\lambda_\alpha$  for  $A_\mu = B_\mu^\alpha$ .

Generic solution of the above KK equation is given by

$$f_n(y) = e^{skR|y|/2} \left[ A_\alpha(m_n) J_\alpha \left( \frac{m_n}{k} e^{kR|y|} \right) + B_\alpha(m_n) Y_\alpha \left( \frac{m_n}{k} e^{kR|y|} \right) \right], \quad (\text{A.4})$$

where  $\alpha = \sqrt{(s/2)^2 + M_\Phi^2/k^2}$ , and  $A_\alpha, B_\alpha$  are determined by the boundary conditions at  $y = 0, \pi$ . To utilize those boundary conditions, it is convenient to introduce the following functions:

$$\begin{aligned} f_{J_{0-}}(q) &= J_\alpha \left( \frac{q}{k} \right), & f_{J_{0+}}(q) &= \left[ \left( r_0 - \frac{s}{2} \right) J_\alpha \left( \frac{q}{k} \right) - \frac{q}{k} J'_\alpha \left( \frac{q}{k} \right) \right], \\ f_{Y_{0-}}(q) &= Y_\alpha \left( \frac{q}{k} \right), & f_{Y_{0+}}(q) &= \left[ \left( r_0 - \frac{s}{2} \right) Y_\alpha \left( \frac{q}{k} \right) - \frac{q}{k} Y'_\alpha \left( \frac{q}{k} \right) \right], \\ f_{J_{\pi-}}(q) &= J_\alpha \left( \frac{q}{T} \right), & f_{J_{\pi+}}(q) &= \left[ \left( \frac{s}{2} - r_\pi \right) J_\alpha \left( \frac{q}{T} \right) + \frac{q}{T} J'_\alpha \left( \frac{q}{T} \right) \right], \\ f_{Y_{\pi-}}(q) &= Y_\alpha \left( \frac{q}{T} \right), & f_{Y_{\pi+}}(q) &= \left[ \left( \frac{s}{2} - r_\pi \right) Y_\alpha \left( \frac{q}{T} \right) + \frac{q}{T} Y'_\alpha \left( \frac{q}{T} \right) \right], \end{aligned} \quad (\text{A.5})$$

where

$$T = k e^{-\pi k R}, \quad r_0 k = \{m_S, -M_F, M_F, m_V\}, \quad r_\pi k = \{\tilde{m}_S, -M_F, M_F, \tilde{m}_V\} \quad (\text{A.6})$$

for the boundary masses  $m_S, \tilde{m}_S$  of  $\phi$  at  $y = 0, \pi$  and the boundary masses  $m_V, \tilde{m}_V$  of  $A_\mu$  at  $y = 0, \pi$ . Again, we are using the mass parameter convention defined in (34) and (35). Explicitly,  $m_S = m$  and  $\tilde{m}_S = \tilde{m}$  for  $\phi = \varphi$ ,  $m_S = \tilde{m}_S = 2k$  for  $\phi = A_5^\sigma$ ,  $m_S = \tilde{m}_S = 0$  for  $\phi = c_A^\sigma, c_B^\alpha$ , and the boundary masses of vector field are defined as

$$- \int d^4x dy \sqrt{-G} \left( \frac{m_V}{g_5^2} \frac{\delta(y)}{\sqrt{G_{55}}} - \frac{\tilde{m}_V}{g_5^2} \frac{\delta(y - \pi)}{\sqrt{G_{55}}} \right) G^{MN} A_M A_N, \quad (\text{A.7})$$

which gives

$$- \int d^4x dy \frac{e^{-2kR|y|}}{R} (m_V \delta(y) - \tilde{m}_V \delta(y - \pi)) \eta^{\mu\nu} A_\mu A_\nu \quad (\text{A.8})$$

after the field redefinition (38).

Imposing the orbifold parity conditions  $\Phi(-y) = z\Phi(y)$  and  $\Phi(-y + \pi) = z'\Phi(y + \pi)$  gives rise to the constraint:

$$\begin{pmatrix} f_{J_0 z}(m_n) & f_{Y_0 z}(m_n) \\ f_{J_\pi z'}(m_n) & f_{Y_\pi z'}(m_n) \end{pmatrix} \begin{pmatrix} A_\alpha \\ B_\alpha \end{pmatrix} = 0. \quad (\text{A.9})$$

This constraint can be used to determine the KK spectrum  $\{m_n\}$ , yielding

$$f_{J_0 z}(m_n) f_{Y_\pi z'}(m_n) - f_{Y_0 z}(m_n) f_{J_\pi z'}(m_n) = 0. \quad (\text{A.10})$$

This then implies that the KK spectrum corresponds to the zeros of

$$N_{zz'}^\Phi = \pi k^{z/2} T^{z'/2} (f_{J_0 z}(q) f_{Y_\pi z'}(q) - f_{Y_0 z}(q) f_{J_\pi z'}(q)), \quad (\text{A.11})$$

where the prefactor  $\pi k^{z/2} T^{z'/2}$  is introduced to achieve the asymptotic behavior

$$\frac{1}{2} \ln N_{zz'}^\Phi(i|q|) = iB_\Phi |q| + A_\Phi \ln |q| + \mathcal{O}(|q|^{-1}) \quad \text{at } |q| \rightarrow \infty.$$

One can confirm that  $N_{zz'}^\Phi(q)$  is a holomorphic even function on the complex plane of  $q$ .

For the computation of 1-loop gauge couplings, we do not need the full expression of the  $N$ -function, but the asymptotic behaviors in the limits  $|q| \rightarrow 0, \infty$ . It is straightforward to find that

$$N_{zz'}^\Phi(q) = 2q^{\frac{z+z'}{2}} \cos \left( \frac{q(e^{\pi k R} - 1)}{k} + \frac{z+z'}{4} \pi \right) + \mathcal{O}(q^{-2}) \quad \text{at } |q| \rightarrow \infty,$$

from which we find

$$A_\Phi = \frac{z+z'}{4}, \quad iB_\Phi = \frac{e^{\pi k R} - 1}{2k} \quad \text{for } \Phi = \{\phi, \psi_L, \psi_R, A_\mu\}.$$

Note that the asymptotic form of  $N_{zz'}^\Phi$  at  $|q| \rightarrow \infty$  is independent of  $\alpha, r_0$  and  $r_\pi$ , and therefore independent of the bulk and boundary masses of the gauge and matter fields in the model. It is determined just by the orbifold parities of  $\Phi$  and the background geometry, i.e.  $k$  and  $R$ , which affects the KK spectral density at  $m_n \rightarrow \infty$ . Note also that

$$N_{zz'}^\psi = N_{zz'}^{\psi_L} N_{\bar{z}\bar{z}'}^{\psi_R} \quad (\bar{z} = -z, \bar{z}' = -z'),$$

and thus

$$A_\psi = A_{\psi_L} + A_{\psi_R} = 0,$$

which means that 5D Dirac fermion does not give rise to a logarithmic divergence.

As we have noticed, most of the model-parameter dependence of 1-loop gauge couplings is determined by the behavior of  $N_{zz'}^\Phi$ , in the limit  $|q| \rightarrow 0$ . Specifically it is determined by

$$N_{zz'}^\Phi = (-q^2)^{n_0^\Phi} (N_0^\Phi + \mathcal{O}(q^2/m_{KK}^2)) \quad \text{for } |q| \ll m_{KK}, \quad (\text{A.12})$$

where  $n_0^\Phi$  is the number of zero mode from  $\Phi$ , and  $m_{KK}$  is the lightest KK mass. In our convention,  $n_0^\Phi = 0$  or 1. For an explicit expression of  $N_0^\Phi$ , let us introduce

$$Q_{zz'}^{(u)}(x) = \frac{1}{x} \left( E_{zz'}^{(u)}(x) - E_{zz'}^{(u)}(-x) \right), \quad (\text{A.13})$$

where

$$E_{zz'}^{(u)}(\alpha) = k^{z/2} T^{z'/2} e^{\alpha \pi k R} \left( \alpha - \frac{s+u}{2} + r_0 \right)^{\frac{z+1}{2}} \left( \alpha + \frac{s+u}{2} - r_\pi \right)^{\frac{z'+1}{2}}$$

with the convention that  $(\alpha - \frac{s+u}{2} + r_0)^{(z+1)/2} = 1$  for  $\alpha - \frac{s+u}{2} + r_0 = 0, z+1=0$  and also  $(\alpha + \frac{s+u}{2} - r_\pi)^{(z'+1)/2} = 1$  for  $\alpha + \frac{s+u}{2} - r_\pi = 0, z'+1=0$ . We then find

$$N_{zz'}^\Phi(q) = Q_{zz'}^\Phi - R_{zz'}^\Phi q^2 + \mathcal{O}(q^4/m_{KK}^2) \quad \text{at } |q| \rightarrow 0, \quad (\text{A.14})$$

where

$$\begin{aligned} Q_{zz'}^\Phi &= Q_{zz'}^{(0)}(\alpha), \\ R_{zz'}^\Phi &= \frac{e^{\pi k R}}{4\alpha k^2} \left( Q_{zz'}^{(2)}(\alpha+1) - Q_{zz'}^{(2)}(\alpha-1) \right). \end{aligned} \quad (\text{A.15})$$

With the above results, one immediately finds that  $\Phi$  does not have a zero mode in case with  $Q_{zz'}^\Phi \neq 0$ , and then

$$N_0^\Phi = Q_{zz'}^\Phi. \quad (\text{A.16})$$

On the other hand, in other case with  $Q_{zz'}^\Phi = 0$ , there is a zero mode, and

$$N_0^\Phi = R_{zz'}^\Phi. \quad (\text{A.17})$$

Let us derive an explicit form of  $Q_{zz'}^\Phi$  and  $R_{zz'}^\Phi$  in some simple cases. For  $A_{\mu++}$  with  $M_V = m_V = \tilde{m}_V = 0$ ,  $\psi_L$  with  $M_F \neq 0$ , and  $\phi$  with  $M_S \neq 0$  and  $m_S = \tilde{m}_S = 0$ , we find

$$\begin{aligned} Q_{++}^{A_\mu} &= 0, \quad N_0^{A_{\mu++}} = R_{++}^{A_\mu} = 2\pi R e^{\pi k R/2}, \\ Q_{++}^{\psi_L} &= 0, \quad N_0^{\psi_{L++}} = R_{++}^{\psi_L} = 2e^{\pi k R/2} \left( \frac{\sinh(M_F - k/2)\pi R}{M_F - k/2} \right), \\ N_0^{\psi_{L+-}} &= Q_{+-}^{\psi_L} = 2e^{-M_F \pi R}, \quad N_0^{\psi_{L-+}} = Q_{-+}^{\psi_L} = 2e^{M_F \pi R}, \\ N_0^{\phi_{++}} &= Q_{++}^\phi = \frac{2M_S^2 e^{-\pi k R/2}}{\alpha_S k} \sinh(\alpha_S \pi k R), \\ N_0^{\phi_{\pm\mp}} &= Q_{\pm\mp}^\phi = \frac{2e^{\pm \pi k R/2}}{\alpha_S} \left[ \alpha_S \cosh(\alpha_S \pi k R) \mp 2 \sinh(\alpha_S \pi k R) \right], \\ N_0^{\phi_{--}} &= Q_{--}^\phi = \frac{2e^{\pi k R/2}}{\alpha_S k} \sinh(\alpha_S \pi k R), \end{aligned}$$

where  $\alpha_S = \sqrt{4 + M_S^2/k^2}$ .

Let us now consider the  $N$ -function in more general case that there is a mass mixing between  $\Phi$ 's with *different* orbifold parities. In such case, the  $N$ -function takes a more complicate form as the mass eigenstate does not have a definite orbifold parity. Let  $\{\Phi_I\}$  ( $I = 1, 2, \dots, n_\Phi$ ) denote a set of 5D fields with the same 4D spin and unbroken gauge charges in the orbifold parity eigenbasis, and  $\{\Phi_A\}$  ( $A = 1, 2, \dots, n_\Phi$ ) denote the same set of fields, but in the bulk mass eigenbasis which is related to the parity eigenbasis by a unitary rotation:

$$\Phi_A = \sum_I U_{AI} \Phi_I. \quad (\text{A.18})$$

Here each fermionic  $\Phi_I$  is either a left-handed spinor ( $\psi_L$ ) or a right-handed spinor ( $\psi_R$ ). The KK wavefunction in the decomposition

$$\Phi_A(x, y) = \sum_n \Phi_{An}(x) f_{An}(y)$$

satisfies

$$[-e^{skR|y|} \partial_y (e^{-skR|y|} \partial_y) + R^2 M_A^2] f_{An} = R^2 e^{2kR|y|} m_n^2 f_{An}, \quad (\text{A.19})$$

where again

$$s = \{4, 1, 1, 2\}, \quad M_A^2 = \{M_{SA}^2, M_{FA}(M_{FA} + k), M_{FA}(M_{FA} - k), M_{VA}^2\} \quad (\text{A.20})$$

for  $\Phi_A = \{\phi_A, e^{-2kR|y|} \psi_{AL}, e^{-2kR|y|} \psi_{AR}, A_\mu^A\}$ . As the orbifold boundary conditions are defined in the basis  $\{\Phi_I\}$ , it is more nontrivial to find the resulting constraints on the KK spectrum and the corresponding  $N$ -functions. It turns out that the  $N$ -function in the presence of mass mixing can be constructed with the following functions:

$$f_{J_0 z_I}^{IA}(q), f_{Y_0 z_I}^{IA}(q), f_{J_\pi z_I'}^{IA}(q), f_{Y_\pi z_I'}^{IA}(q), \quad (\text{A.21})$$

where  $z_I, z_I'$  are the orbifold parities of  $\Phi_I$  and

$$\begin{aligned} f_{J_0-}^{IA}(q) &= J_{\alpha_A} \left( \frac{q}{k} \right), & f_{J_0+}^{IA}(q) &= \left[ \left( r_{0IA} - \frac{s}{2} \right) J_{\alpha_A} \left( \frac{q}{k} \right) - \frac{q}{k} J'_{\alpha_A} \left( \frac{q}{k} \right) \right] \\ f_{Y_0-}^{IA}(q) &= Y_{\alpha_A} \left( \frac{q}{k} \right), & f_{Y_0+}^{IA}(q) &= \left[ \left( r_{0IA} - \frac{s}{2} \right) Y_{\alpha_A} \left( \frac{q}{k} \right) - \frac{q}{k} Y'_{\alpha_A} \left( \frac{q}{k} \right) \right] \\ f_{J_\pi-}^{IA}(q) &= J_{\alpha_A} \left( \frac{q}{T} \right), & f_{J_\pi+}^{IA}(q) &= \left[ \left( \frac{s}{2} - r_{\pi IA} \right) J_{\alpha_A} \left( \frac{q}{T} \right) + \frac{q}{T} J'_{\alpha_A} \left( \frac{q}{T} \right) \right] \\ f_{Y_\pi-}^{IA}(q) &= Y_{\alpha_A} \left( \frac{q}{T} \right), & f_{Y_\pi+}^{IA}(q) &= \left[ \left( \frac{s}{2} - r_{\pi IA} \right) Y_{\alpha_A} \left( \frac{q}{T} \right) + \frac{q}{T} Y'_{\alpha_A} \left( \frac{q}{T} \right) \right]. \end{aligned}$$

Here

$$\alpha_A = \sqrt{(s/2)^2 + M_A^2/k^2},$$

and

$$\begin{aligned} kr_{0IA} &= \sum_J \frac{(m_{S,V})_{IJ} U_{AJ}^*}{U_{AI}^*}, & kr_{\pi IA} &= \sum_J \frac{(\tilde{m}_{S,V})_{IJ} U_{AJ}^*}{U_{AI}^*} \quad \text{for } \Phi_I = \phi_I, A_\mu^I, \\ kr_{0IA} &= \mp M_{FA}, & kr_{\pi IA} &= \mp M_{FA} \quad \text{for } \Phi_I = \psi_{IL}, \psi_{IR}, \end{aligned} \quad (\text{A.22})$$

where  $(m_{S,V})_{IJ}$  and  $(\tilde{m}_{S,V})_{IJ}$  are the boundary mass matrices of  $\phi_I, A_\mu^I$  at  $y = 0$  and  $y = \pi$ , respectively, defined in the orbifold parity eigenbasis.



Then, for the KK wavefunction

$$f_{An}(y) = e^{skR|y|/2} \left[ A_A(m_n) J_\alpha \left( \frac{m_n}{k} e^{kR|y|} \right) + B_A(m_n) Y_\alpha \left( \frac{m_n}{k} e^{kR|y|} \right) \right],$$

the orbifold boundary conditions yield

$$\mathcal{B} \begin{pmatrix} A_A \\ B_A \end{pmatrix} = \begin{pmatrix} B_{J_0} & B_{Y_0} \\ B_{J_\pi} & B_{Y_\pi} \end{pmatrix} \begin{pmatrix} A_A \\ B_A \end{pmatrix} = 0, \quad (\text{A.23})$$

where  $\mathcal{B}$  is a  $2n_\Phi \times 2n_\Phi$  matrix given by

$$\begin{aligned} (B_{J_0})_{IA} &= U_{AI}^* f_{J_0 z_I}^{IA}, & (B_{Y_0})_{IA} &= U_{AI}^* f_{Y_0 z_I}^{IA}, \\ (B_{J_\pi})_{IA} &= U_{AI}^* f_{J_\pi z'_I}^{IA}, & (B_{Y_\pi})_{IA} &= U_{AI}^* f_{Y_\pi z'_I}^{IA} \quad \text{for } \Phi_I = \phi_I \text{ or } A_\mu^I, \end{aligned}$$

and

$$\begin{aligned} (B_{J_0})_{IA} &= (U F_0^{L,R})_{AI}^* f_{J_0 z_I}^{IA}, & (B_{Y_0})_{IA} &= (U F_0^{L,R})_{AI}^* f_{Y_0 z_I}^{IA}, \\ (B_{J_\pi})_{IA} &= (U F_\pi^{L,R})_{AI}^* f_{J_\pi z'_I}^{IA}, & (B_{Y_\pi})_{IA} &= (U F_\pi^{L,R})_{AI}^* f_{Y_\pi z'_I}^{IA} \quad \text{for } \Phi_I = \psi_{IL}, \psi_{IR}, \end{aligned}$$

where

$$\begin{aligned} (F_{0,\pi}^{L,R})_{IJ} &= \frac{(\mathcal{F}_{0,\pi}^{L,R})_{IJ}}{[\det(\mathcal{F}_{0,\pi}^{L,R})]^{1/n_\Phi}}, \\ (\mathcal{F}_0^{L,R})_{IJ} &= \delta_{IJ} \pm z_I \mu_{IJ}, \quad (\mathcal{F}_\pi^{L,R})_{IJ} = \delta_{IJ} \pm z'_I \tilde{\mu}_{IJ} \end{aligned}$$

for the boundary fermion masses  $\mu_{IJ}, \tilde{\mu}_{IJ}$  defined in (35).

With (A.23), the  $N$  function of  $\{\Phi_I\}$  is proportional to the determinant of the  $2n_\Phi \times 2n_\Phi$  matrix  $\mathcal{B}$ . In fact, one can show that the  $N$ -function can be reduced to the determinant of an  $n_\Phi \times n_\Phi$  matrix:

$$N^{\{\Phi\}}(q) = \det(B_N(q)) \quad (\text{A.24})$$

where

$$\begin{aligned} (B_N)_{IJ} &= \sum_A U_{AI}^* U_{AJ} N_{z_I z'_J}^{IJ,A}(q) \quad \text{for } \Phi_I = \phi_I \text{ or } A_\mu^I, \\ (B_N)_{IJ} &= \sum_A (U F_0^{L,R})_{AI}^* (U F_\pi^{L,R})_{AJ} N_{z_I z'_J}^{IJ,A}(q) \quad \text{for } \Phi_I = \psi_{IL}, \psi_{IR} \end{aligned} \quad (\text{A.25})$$

with

$$N_{z_I z'_J}^{IJ,A}(q) = \pi k^{z_I/2} T^{z'_J/2} \left( f_{J_0 z_I}^{IA}(q) f_{Y_\pi z'_J}^{JA*}(q) - f_{Y_0 z_I}^{IA}(q) f_{J_\pi z'_J}^{JA*}(q) \right). \quad (\text{A.26})$$

Note that this function is nothing but the  $N$  function defined in (A.11) with  $\alpha \rightarrow \alpha_A$ ,  $r_0 \rightarrow r_{0IA}$ ,  $r_\pi \rightarrow r_{\pi IA}$  and  $z, z' \rightarrow z_I, z'_I$ . Furthermore its limiting behavior at  $|q| \rightarrow \infty$  is independent of  $\alpha_A, r_{0IA}, r_{\pi IA}$ :

$$\frac{1}{2} \ln N_{z_I z'_J}^{IJ,A}(i|q|) = \frac{e^{\pi k R} - 1}{2k} |q| + \frac{z_I + z'_I}{4} \ln |q| + \mathcal{O}(|q|^{-1}),$$

and therefore

$$\frac{1}{2} \ln N^{\{\Phi\}}(i|q|) = \frac{n_\Phi (e^{\pi k R} - 1)}{2k} |q| + \sum_I \frac{(z_I + z'_I)}{4} \ln |q| + \mathcal{O}(|q|^{-1})$$

at  $|q| \rightarrow \infty$ .

To obtain the 1-loop corrections to low energy gauge couplings induced by  $\{\Phi\}$ , we need to know the limiting behavior of  $N^{\{\Phi\}}$  at  $q \rightarrow 0$ :

$$N^{\{\Phi\}} = \det(B_N) = (-q^2)^{n_0^{\{\Phi\}}} \left( N_0^{\{\Phi\}} + \mathcal{O}(q^2/m_{KK}^2) \right).$$

It is straightforward to find  $N_0^{\{\Phi\}}$  from the limiting behavior of  $N_{zz'}^{\Phi}$  in (A.14) and the expression of  $(B_N)_{IJ}$  in (A.25).

## Appendix B. KK thresholds with boundary matter fields

In this paper, we did not include a boundary matter field separately. In fact, boundary matter field can always be considered as a 4D mode of bulk matter field localized at the boundary in the limit that the 5D mass of bulk field approaches to the cutoff scale  $\Lambda$ <sup>¶</sup>. This means that the 1-loop gauge coupling in the presence of boundary matter field can be obtained from our results by taking an appropriate limit. Here we discuss this point with simple examples in flat spacetime background.

Let us first consider a Dirac fermion  $\psi_{++}$  with bulk mass  $M_F$ . By taking the limit  $k \rightarrow 0$  for the result in Table 4, one easily finds that the 1-loop correction due to  $\psi_{++}$  is given by

$$\frac{1}{8\pi^2} \Delta_a^{\psi_{++}} = \frac{1}{12\pi^2} \text{Tr}(T_a^2(\psi)) \left[ \ln \frac{M_F}{p} - \ln(\sinh M_F \pi R) \right]. \quad (\text{B.1})$$

In the limit  $M_F \rightarrow \Lambda \gg 1/R$ , the chiral zero mode becomes localized at  $y = 0$ . On the other hand, all KK modes get a mass comparable to  $\Lambda$ , and therefore can be integrated out while leaving a trace only in the Wilsonian couplings at  $\Lambda$ . Indeed  $\Delta_a^{\psi_{++}}$  in the limit  $M_F \rightarrow \Lambda$  becomes the 1-loop correction due to a 4D boundary chiral fermion after subtracting the power-law divergence which should be absorbed into the renormalization of the 5D gauge coupling  $1/g_{5a}^2$  at  $\Lambda$ :

$$\frac{1}{8\pi^2} \Delta_a^{\psi_{++}} \rightarrow \frac{1}{12\pi^2} \text{Tr}(T_a^2(\psi)) \left[ \ln \frac{\Lambda}{p} - \Lambda \pi R + \mathcal{O}(1) \right]. \quad (\text{B.2})$$

As another example, let us consider  $\psi_{+-}$  with bulk mass  $M_F$ , which gives a correction

$$\frac{1}{8\pi^2} \Delta_a^{\psi_{+-}} = \frac{1}{12\pi^2} \text{Tr}(T_a^2(\psi)) M_F \pi R. \quad (\text{B.3})$$

In the limit  $M_F \rightarrow \Lambda \gg 1/R$ , there appear two chiral fermion modes localized at the boundaries, one at  $y = 0$  and another at  $y = \pi$ , which form a 4D Dirac fermion with 4D mass  $m_D = 2M_F e^{-M_F \pi R}$ , while all other modes have a mass of  $\mathcal{O}(\Lambda)$ . As the above 1-loop gauge coupling assumes that there is no light mode with a mass lighter than the external momentum  $p$  of the gauge boson zero mode, it can be directly used only for  $p < m_D$ . We then find

$$\frac{1}{8\pi^2} \Delta_a^{\psi_{+-}} \rightarrow \frac{1}{12\pi^2} \text{Tr}(T_a^2(\psi)) \left[ 2 \ln \frac{\Lambda}{m_D} - \Lambda \pi R + \mathcal{O}(1) \right], \quad (\text{B.4})$$

<sup>¶</sup> For scalar field, we also need proper boundary masses comparable to  $\Lambda$ .

which corresponds, after subtracting the power law divergence, to the 1-loop threshold due to a massive 4D Dirac fermion with mass  $m_D$ . If we consider the limit that  $m_D$  becomes even smaller than  $p$ , the IR cutoff of the momentum integral of the localized modes should be taken as  $p$ , and then we arrive at the standard 1-loop correction due to a massless 4D Dirac fermion:

$$\frac{1}{8\pi^2}\Delta_a = \frac{1}{6\pi^2}\text{Tr}(T_a^2(\psi))\ln(\Lambda/p). \quad (\text{B.5})$$

Let us finally consider the case of two 5D Dirac fermions  $\psi_{++}^1$  and  $\psi_{--}^2$  with a diagonal 5D mass matrix  $M_{Fpq} = M_{Fp}\delta_{pq}$  ( $p, q = 1, 2$ ) and a boundary mass-mixing

$$\int d^4x dy \delta(y) 2\mu (\bar{\psi}^1 \psi^2 + \bar{\psi}^2 \psi^1). \quad (\text{B.6})$$

(Note that 5D fermion has a mass-dimension 2, and thus  $\mu$  is a dimensionless parameter in our convention.) With the results in Table 6, one easily finds

$$\frac{1}{8\pi^2}\Delta_a^{\{\psi^1, \psi^2\}} = -\frac{1}{12\pi^2}\text{Tr}(T_a^2(\psi))\ln\left[\left(\frac{\mu^2}{1+\mu^2}\right)e^{(M_{F1}-M_{F2})\pi R}\right]. \quad (\text{B.7})$$

In the limit  $M_{F2} \rightarrow -\Lambda$  with  $\mu \ll 1$ ,  $\psi_{--}^2$  gives a chiral zero mode  $\chi$  localized at  $y = 0$ , while all other modes of  $\psi_{--}$  are decoupled with a mass comparable to  $\Lambda$ . The resulting effective theory contains a 5D fermion  $\psi_{++}^1$  and a chiral boundary fermion  $\chi$  with a mass mixing:

$$\int d^4x dy \delta(y) 2\mu_{\text{eff}} (\bar{\psi}^1 \chi + \bar{\chi} \psi^1), \quad (\text{B.8})$$

where  $\mu_{\text{eff}} = 2\mu\sqrt{\Lambda} \ll \sqrt{\Lambda}$  for the canonically normalized 4D fermion  $\chi$ . In the same limit,

$$\frac{1}{8\pi^2}\Delta_a^{\{\psi^1, \psi^2\}} \rightarrow \frac{1}{12\pi^2}\text{Tr}(T_a^2(\psi))\left[\ln\left(\frac{\Lambda}{\mu_{\text{eff}}^2}\right) - \ln(e^{M_{F1}\pi R}) - \Lambda\pi R + \mathcal{O}(1)\right],$$

which corresponds to the 1-loop threshold in the effective theory again after subtracting the power law divergence.

**Table 1.** 1-loop gauge couplings induced by 5D vector fields  $A_M^\sigma$  in the limit  $p \ll m_{KK}$  where  $m_{KK}$  is the lowest nonzero KK mass. Here  $c_a(A) = \text{Tr}(T_a^2(A_M))$ .

| $(zz')$ | $\Delta_a^{\{A\}}$   |
|---------|--|
| $(++)$  | $\frac{c_a(A)}{12} [22\pi kR - 23 \ln(\Lambda\pi R) + 44 \ln(p\pi R)]$   |
| $(+-)$  | $\frac{c_a(A)}{12} (-22\pi kR)$  |
| $(-+)$  | $\frac{c_a(A)}{12} (22\pi kR)$   |
| $(--)$  | $\frac{c_a(A)}{12} [21 \ln\left(\frac{\sinh \pi kR}{\pi kR}\right) - \pi kR + 23 \ln(\Lambda\pi R) - 2 \ln(p\pi R)]$ |

**Table 2.** 1-loop gauge couplings induced by 5D vector fields  $B_M^\alpha$  and Goldstone bosons  $\pi^\alpha$  for the range of  $M_B$  which does not give any zero mode lighter than  $p$ . Here  $c_a(B) = \text{Tr}(T_a^2(B_M))$ ,  $M_B = g_{5\alpha}\lambda_\alpha$ , and  $\alpha_B = \sqrt{1 + M_B^2/k^2}$ , where  $M_B$  is the canonical 5D mass of  $B_M^\alpha$ .

| $(zz')$ | $\Delta_a^{\{B\}}$  |
|---------|---|
| $(++)$  | $\frac{c_a(B)}{12} \left[ 20 \ln\left(\frac{\sinh \alpha_B \pi kR}{\alpha_B \pi kR}\right) + 42 \ln(M_B \pi R) - 22 \ln(\Lambda \pi R) \right]$ |
| $(+-)$  | $\frac{c_a(B)}{12} \left[ 20 \ln\left(\frac{\alpha_B \cosh \alpha_B \pi kR - \sinh \alpha_B \pi kR}{\alpha_B}\right) \right]$                   |
| $(-+)$  | $\frac{c_a(B)}{12} \left[ 20 \ln\left(\frac{\alpha_B \cosh \alpha_B \pi kR + \sinh \alpha_B \pi kR}{\alpha_B}\right) \right]$                   |
| $(--)$  | $\frac{c_a(B)}{12} \left[ 20 \ln\left(\frac{\sinh \alpha_B \pi kR}{\alpha_B \pi kR}\right) - 2 \ln(M_B \pi R) + 22 \ln(\Lambda \pi R) \right]$  |

**Table 3.** 1-loop gauge couplings induced by a 5D real scalar  $\varphi$  with definite orbifold parities  $z, z'$ . Here  $c_a(\varphi) = \text{Tr}(T_a^2(\varphi))$ ,  $\alpha = \sqrt{4 + M_S^2/k^2}$ , where  $M_S$ ,  $m_S$  and  $\tilde{m}_S$  are the bulk and boundary masses of  $\varphi$ .  $\varphi^{(0)}$  denotes a particular type of 5D scalar field with  $(zz') = (++)$ ,  $m_S = \tilde{m}_S$  and  $M_S^2 = m_S(m_S - 4k)$ , which has a zero mode lighter than  $p$ .

| $(zz')$ | $\Delta_a^{\{\varphi\}}$  |
|---------|---|
| $(++)$  | $\frac{-1}{12}c_a(\varphi^{(0)}) \left[ \ln \left( \frac{\sinh(m_S - k)\pi R}{(m_S - k)\pi R} \right) + \pi k R - \ln(\Lambda \pi R) + 2 \ln(p \pi R) \right]$ $\frac{-1}{12}c_a(\varphi) \left[ \ln \left( \frac{\alpha k(m_S - \tilde{m}_S) \cosh \alpha \pi k R + (\alpha^2 k^2 - (2k - m_S)(2k - \tilde{m}_S)) \sinh \alpha \pi k R}{\alpha k} \right) - \ln \Lambda \right]$ |
| $(+-)$  | $\frac{-c_a(\varphi)}{12} \left[ \ln \left( \frac{\alpha k \cosh \alpha \pi k R - (2k - m_S) \sinh \alpha \pi k R}{\alpha k} \right) \right]$   |
| $(-+)$  | $\frac{-c_a(\varphi)}{12} \left[ \ln \left( \frac{\alpha k \cosh \alpha \pi k R + (2k - \tilde{m}_S) \sinh \alpha \pi k R}{\alpha k} \right) \right]$   |
| $(--)$  | $\frac{-c_a(\varphi)}{12} \left[ \ln \left( \frac{\sinh \alpha \pi k R}{\alpha \pi k R} \right) + \ln(\Lambda \pi R) \right]$   |

**Table 4.** 1-loop gauge couplings induced by a Dirac fermion  $\psi$  with definite orbifold parities. Here  $c_a(\psi) = \text{Tr}(T_a^2(\psi))$  and  $M_F$  is the bulk mass of  $\psi$ .

| $(zz')$ | $\Delta_a^{\{\psi\}}$  |
|---------|--|
| $(++)$  | $-\frac{2}{3}c_a(\psi) \left[ \ln \left( \frac{\sinh(M_F - k/2)\pi R}{(M_F - k/2)\pi R} \right) + \frac{1}{2}\pi k R + \ln(p \pi R) \right]$ |
| $(+-)$  | $+\frac{2}{3}c_a(\psi) M_F \pi R$  |
| $(-+)$  | $-\frac{2}{3}c_a(\psi) M_F \pi R$  |
| $(--)$  | $-\frac{2}{3}c_a(\psi) \left[ \ln \left( \frac{\sinh(M_F + k/2)\pi R}{(M_F + k/2)\pi R} \right) + \frac{1}{2}\pi k R + \ln(p \pi R) \right]$ |

**Table 5.** 1-loop corrections induced by two real scalars  $\{\varphi_{z_1 z'_1}^1, \varphi_{z_2 z'_2}^2\}$  which have the same gauge charge, but can have different orbifold parities. Here  $\alpha_i = \sqrt{4 + M_{S_i}^2/k^2}$  ( $i = 1, 2$ ) for the bulk mass eigenvalues  $M_{S_i}$ , and  $s \equiv \sin \theta_S$ ,  $c \equiv \cos \theta_S$  for the mixing angle  $\theta_S$ . We are considering a generic bulk mass matrix which does not give any zero mode lighter than  $p$ , while the boundary masses are assumed to be zero for simplicity.

| $(z_1 z'_1)$<br>$(z_2 z'_2)$ | $\Delta_a^{\{\varphi^1, \varphi^2\}}$   |
|------------------------------|---|
| $(++)$<br>$(++)$             | $\frac{-c_a(\varphi)}{12} \left[ \ln \left( \frac{\sinh \alpha_1 \pi k R}{\alpha_1 \pi k R} \right) \left( \frac{\sinh \alpha_2 \pi k R}{\alpha_2 \pi k R} \right) + 2 \ln(M_{S1} \pi R)(M_{S2} \pi R) - 2 \ln(\Lambda \pi R) \right]$  |
| $(++)$<br>$(+-)$             | $\frac{-c_a(\varphi)}{12} \left[ \ln \left\{ c^2 \left( \frac{(M_{S1} \pi R)^2 \sinh \alpha_1 \pi k R}{\alpha_1 \pi k R} \right) \left( \frac{\alpha_2 \cosh \alpha_2 \pi k R - 2 \sinh \alpha_2 \pi k R}{\alpha_2} \right) \right. \right.$<br>$\left. \left. + s^2 \left( \frac{(M_{S2} \pi R)^2 \sinh \alpha_2 \pi k R}{\alpha_2 \pi k R} \right) \left( \frac{\alpha_1 \cosh \alpha_1 \pi k R - 2 \sinh \alpha_1 \pi k R}{\alpha_1} \right) \right\} - \ln(\Lambda \pi R) \right]$  |
| $(++)$<br>$(-+)$             | $\frac{-c_a(\varphi)}{12} \left[ \ln \left\{ c^2 \left( \frac{(M_{S1} \pi R)^2 \sinh \alpha_1 \pi k R}{\alpha_1 \pi k R} \right) \left( \frac{\alpha_2 \cosh \alpha_2 \pi k R + 2 \sinh \alpha_2 \pi k R}{\alpha_2} \right) \right. \right.$<br>$\left. \left. + s^2 \left( \frac{(M_{S2} \pi R)^2 \sinh \alpha_2 \pi k R}{\alpha_2 \pi k R} \right) \left( \frac{\alpha_1 \cosh \alpha_1 \pi k R + 2 \sinh \alpha_1 \pi k R}{\alpha_1} \right) \right\} - \ln(\Lambda \pi R) \right]$  |
| $(++)$<br>$(--)$             | $\frac{-c_a(\varphi)}{12} \ln \left\{ ((\alpha_1 c^2 + \alpha_2 s^2)^2 - 4) \left( \frac{\sinh \alpha_1 \pi k R}{\alpha_1} \frac{\sinh \alpha_2 \pi k R}{\alpha_2} \right) + 4c^2 s^2 \sinh^2 \frac{(\alpha_1 - \alpha_2) \pi k R}{2} \right\}$   |
| $(+-)$<br>$(+-)$             | $\frac{-c_a(\varphi)}{12} \left[ \ln \left( \frac{\alpha_1 \cosh \alpha_1 \pi k R - 2 \sinh \alpha_1 \pi k R}{\alpha_1} \right) + \ln \left( \frac{\alpha_2 \cosh \alpha_2 \pi k R - 2 \sinh \alpha_2 \pi k R}{\alpha_2} \right) \right]$   |
| $(+-)$<br>$(-+)$             | $\frac{-c_a(\varphi)}{12} \ln \left\{ \left( \frac{\alpha_1 \cosh \alpha_1 \pi k R - 2 \sinh \alpha_1 \pi k R}{\alpha_1} \right) \left( \frac{\alpha_2 \cosh \alpha_2 \pi k R + 2 \sinh \alpha_2 \pi k R}{\alpha_2} \right) \right.$<br>$\left. + 4s^2 \left( \frac{\alpha_2 \sinh \alpha_1 \pi k R \cosh \alpha_2 \pi k R - \alpha_1 \sinh \alpha_2 \pi k R \cosh \alpha_1 \pi k R}{\alpha_1 \alpha_2} \right) \right.$<br>$\left. + (\alpha_1 - \alpha_2)^2 c^2 s^2 \left( \frac{\sinh \alpha_1 \pi k R}{\alpha_1} \frac{\sinh \alpha_2 \pi k R}{\alpha_2} \right) - 4c^2 s^2 \sinh^2 \frac{(\alpha_1 - \alpha_2) \pi k R}{2} \right\}$ |
| $(+-)$<br>$(--)$             | $\frac{-c_a(\varphi)}{12} \left[ \ln \left\{ c^2 \left( \frac{\alpha_1 \cosh \alpha_1 \pi k R - 2 \sinh \alpha_1 \pi k R}{\alpha_1} \right) \left( \frac{\sinh \alpha_2 \pi k R}{\alpha_2 \pi k R} \right) \right. \right.$<br>$\left. \left. + s^2 \left( \frac{\alpha_2 \cosh \alpha_2 \pi k R - 2 \sinh \alpha_2 \pi k R}{\alpha_2} \right) \left( \frac{\sinh \alpha_1 \pi k R}{\alpha_1 \pi k R} \right) \right\} + \ln(\Lambda \pi R) \right]$  |
| $(-+)$<br>$(-+)$             | $\frac{-c_a(\varphi)}{12} \left[ \ln \left( \frac{\alpha_1 \cosh \alpha_1 \pi k R + 2 \sinh \alpha_1 \pi k R}{\alpha_1} \right) + \ln \left( \frac{\alpha_2 \cosh \alpha_2 \pi k R + 2 \sinh \alpha_2 \pi k R}{\alpha_2} \right) \right]$   |
| $(-+)$<br>$(--)$             | $\frac{-c_a(\varphi)}{12} \left[ \ln \left\{ c^2 \left( \frac{\alpha_1 \cosh \alpha_1 \pi k R + 2 \sinh \alpha_1 \pi k R}{\alpha_1} \right) \left( \frac{\sinh \alpha_2 \pi k R}{\alpha_2 \pi k R} \right) \right. \right.$<br>$\left. \left. + s^2 \left( \frac{\alpha_2 \cosh \alpha_2 \pi k R + 2 \sinh \alpha_2 \pi k R}{\alpha_2} \right) \left( \frac{\sinh \alpha_1 \pi k R}{\alpha_1 \pi k R} \right) \right\} + \ln(\Lambda \pi R) \right]$  |
| $(--)$<br>$(--)$             | $\frac{-c_a(\varphi)}{12} \left[ \ln \left( \frac{\sinh \alpha_1 \pi k R}{\alpha_1 \pi k R} \right) \left( \frac{\sinh \alpha_2 \pi k R}{\alpha_2 \pi k R} \right) + 2 \ln(\Lambda \pi R) \right]$  |

**Table 6.** 1-loop corrections induced by two Dirac fermions  $\{\psi_{z_1 z'_1}^1, \psi_{z_2 z'_2}^2\}$  which have the same gauge charge, but can have different orbifold parities. Here  $M_{F_p}$  ( $p = 1, 2$ ) are the bulk mass eigenvalues, and  $c_{0,\pi}, s_{0,\pi}$  are defined in (71) in terms of the bulk mixing angle and the boundary mass mixings. We are considering a generic parameter range in which all nonzero KK masses are heavier than  $p$ .

| $(z_1 z'_1)$<br>$(z_2 z'_2)$ | $\Delta_a^{\{\psi^1, \psi^2\}}$   |
|------------------------------|---|
| $(++)$<br>$(++)$             | $-\frac{2}{3}c_a(\psi) \left[ \ln \left( \frac{\sinh(M_{F1}-k/2)\pi R}{(M_{F1}-k/2)\pi R} \right) + \ln \left( \frac{\sinh(M_{F2}-k/2)\pi R}{(M_{F2}-k/2)\pi R} \right) + \pi k R + 2 \ln(p\pi R) \right]$  |
| $(++)$<br>$(+-)$             | $-\frac{2}{3}c_a(\psi) \left[ \ln \left\{  c_\pi ^2 \left( \frac{\sinh(M_{F1}-k/2)\pi R}{(M_{F1}-k/2)\pi R} \right) e^{-M_{F2}\pi R} \right. \right.$<br>$\left. \left. +  s_\pi ^2 \left( \frac{\sinh(M_{F2}-k/2)\pi R}{(M_{F2}-k/2)\pi R} \right) e^{-M_{F1}\pi R} \right\} + \frac{1}{2}\pi k R + \ln(p\pi R) \right]$ |
| $(++)$<br>$(-+)$             | $-\frac{2}{3}c_a(\psi) \left[ \ln \left\{  c_0 ^2 \left( \frac{\sinh(M_{F1}-k/2)\pi R}{(M_{F1}-k/2)\pi R} \right) e^{M_{F2}\pi R} \right. \right.$<br>$\left. \left. +  s_0 ^2 \left( \frac{\sinh(M_{F2}-k/2)\pi R}{(M_{F2}-k/2)\pi R} \right) e^{M_{F1}\pi R} \right\} + \frac{1}{2}\pi k R + \ln(p\pi R) \right]$       |
| $(++)$<br>$(--)$             | $-\frac{2}{3}c_a(\psi) \ln  c_0 s_\pi e^{-(M_{F1}-M_{F2})\pi R/2} - c_\pi s_0 e^{(M_{F1}-M_{F2})\pi R/2} ^2$  |
| $(+-)$<br>$(+-)$             | $+\frac{2}{3}c_a(\psi)(M_{F1}\pi R + M_{F2}\pi R)$  |
| $(+-)$<br>$(-+)$             | $-\frac{2}{3}c_a(\psi) \ln  c_0 c_\pi^* e^{-(M_{F1}-M_{F2})\pi R/2} + s_0 s_\pi^* e^{(M_{F1}-M_{F2})\pi R/2} ^2$  |
| $(+-)$<br>$(--)$             | $-\frac{2}{3}c_a(\psi) \left[ \ln \left\{  c_0 ^2 \left( \frac{\sinh(M_{F2}+k/2)\pi R}{(M_{F2}+k/2)\pi R} \right) e^{-M_{F1}\pi R} \right. \right.$<br>$\left. \left. +  s_0 ^2 \left( \frac{\sinh(M_{F1}+k/2)\pi R}{(M_{F1}+k/2)\pi R} \right) e^{-M_{F2}\pi R} \right\} + \frac{1}{2}\pi k R + \ln(p\pi R) \right]$     |
| $(-+)$<br>$(-+)$             | $-\frac{2}{3}c_a(\psi)(M_{F1}\pi R + M_{F2}\pi R)$  |
| $(-+)$<br>$(--)$             | $-\frac{2}{3}c_a(\psi) \left[ \ln \left\{  c_\pi ^2 \left( \frac{\sinh(M_{F2}+k/2)\pi R}{(M_{F2}+k/2)\pi R} \right) e^{M_{F1}\pi R} \right. \right.$<br>$\left. \left. +  s_\pi ^2 \left( \frac{\sinh(M_{F1}+k/2)\pi R}{(M_{F1}+k/2)\pi R} \right) e^{M_{F2}\pi R} \right\} + \frac{1}{2}\pi k R + \ln(p\pi R) \right]$   |
| $(--)$<br>$(--)$             | $-\frac{2}{3}c_a(\psi) \left[ \ln \left( \frac{\sinh(M_{F1}+k/2)\pi R}{(M_{F1}+k/2)\pi R} \right) + \ln \left( \frac{\sinh(M_{F2}+k/2)\pi R}{(M_{F2}+k/2)\pi R} \right) + \pi k R + 2 \ln(p\pi R) \right]$  |

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